The general Boson-Fermion filter as a generalization of the Belavkin quantum filter based on the Hudson-Parthasarathy quantum stochastic calculus

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Abstract

This manuscript presents an advanced generalization of the Belavkin quantum filter, applying it to Boson and Fermion noise via the Hudson-Parthasarathy quantum stochastic calculus. It focuses on estimating quantum system observables and states in noisy conditions, highlighting the transition from non-commutative to commutative observables post-filtering. The work introduces an algorithm for constructing quantum filter coefficients by solving an infinite series of linear algebraic equations, enabling real-time filtering in the observable domain. This development offers significant insights into quantum noise analysis and state estimation, with potential applications in quantum computing and information processing. The manuscript also explores the integration of classical noisy measurements with quantum non-demolition measurements, creating a hybrid estimation framework that enhances quantum filtering capabilities. The work has potential applications in estimating the time varying state of a quantum gravitational wave from measurements on electromagnetic noise fields with which the wave interacts.

Keywords: Quantum Filtering, Boson-Fermion Noise, Hudson-Parthasarathy.

1 Introduction

By measuring generalized Boson noise and Fermion counting processes as a non-demolition measurement, we can estimate evolving observables and system states of a noisy quantum system. Although the original observables will be non-commutative, after filtering/estimation, they will be all commutative. The Hamiltonian and Lindblad operators that describe noisy quantum evolution can be functions of a classical Markovian parameter $\phi(t)$ and we then estimate on a real time basis both the observables and these parameters. The fact that conditioning on the non-demolition measurements restores commutativity is a striking example of how one can generate an evolving family of commuting observable estimates from a system of non-commuting observables. One can then even attempt to calculate bounds on the estimation error average energy between the noisy Heisenberg observables and their estimates in a coherent state and that would give us a clue regarding approximating evolving systems of non-commuting variables using commuting variables. In this paper, we additionally assume that the system operators that describe the Hamiltonian and Lindlbad operators that couple the system to the bath, in addition, depend upon a classical parameter that describes a classical Markov process and that apart from the quantum non-demolition measurement considered above, we also take classical noisy measurements on a function of the Markovian parameter with measurement noise being a non-Gaussian white noise, ie, the time derivative of a Levy process and describe an algorithm for constructing the Abelian family of coefficients of the quantum filter by solving an infinite sequence of linear algebraic equations. The result is a real time quantum filter in observable domain wherein the observable space is the tensor product of the Banach space of bounded Borel functions on classical parameter space with the Banach space of bounded self-adjoint operators in the system Hilbert space.

The generalized Bosonic processes in the Hudson-Parthasarathy quantum stochastic calculus are $\Lambda_b^a(t), N \geq a, b \geq 0, \Lambda_0^0 = t$. The supersymmetric process differentials are $\tilde{\xi}_b^a(t) = (-1)^{\sigma(a,b)\tilde{\Lambda}(t)} d\tilde{\Lambda}_b^a(t), M \geq a, b \geq 0$ where $\tilde{\Lambda}(t) = \sum_{k=r}^{N} \tilde{\Lambda}_k^k(t)$. The processes $\Lambda_b^a(t)$ live in the Boson Fock space $\Gamma_s(\mathcal{H}_1)$ while the supersymmetric processes $\tilde{\xi}_b^a(t)$ live in the Boson Fock space $\Gamma_s(\mathcal{H}_2)$. Both these processes therefore live in

$$\Gamma_s(\mathcal{H}_1) \otimes \Gamma_s(\mathcal{H}_2) = \Gamma_s(\mathcal{H}_1 \oplus \mathcal{H}_2)$$

and hence these two processes mutually commute. Note that

$$[\Lambda_b^a(t), \tilde{\Lambda}_d^c(s)] = 0 \forall a, b, c, d, t, s$$

The quantum Ito formulae are

$$d\Lambda_b^a . d\Lambda_d^c = \epsilon_d^a d\Lambda_a^c,$$

$$d\tilde{\Lambda}_b^a . d\tilde{\Lambda}_d^c = \epsilon_d^a d\tilde{\Lambda}_a^c,$$

and hence,

$$d\tilde{\xi}_b^a(t) . d\tilde{\xi}_d^c(t) = \epsilon_d^a . d\tilde{\xi}_b^c(t)$$

It should be noted that the ξ_b^a satisfy the supercommutation relations:

$$d\xi_b^a(t) . d\xi_d^c(s) - (-1)^{\sigma(a,b)\sigma(c,d)} d\xi_d^c(s) . d\xi_b^a(t) = 0, s \neq t$$

in contrast to the commutation relations satisfied by the Λ^a_b:

$$d\Lambda^a_b(t).d\Lambda^c_d(s) - d\Lambda^c_d(s).d\Lambda^a_b(t) = 0, s \neq t$$

The HPS-qsde is

$$dU(t) = (L^a_b(\phi(t))d\Lambda^b_a(t) + M^a_b(\phi(t))d\tilde{\xi}^b_a(t))U(t)$$

where $\phi(t)$ is a Markovian parameter. It should be noted that the grading index $\sigma(a,b) = 0$ when either $1 \leq a, b \leq r$ and $r+1 \leq a, b \leq N$ and $\sigma(a,b) = -1$ otherwise.

The input measurement process is taken as

$$dY_i(t) = c(a,b)d\Lambda^a_b(t) + d(a)d\tilde{\Lambda}^a_a(t)$$

or equivalently,

$$Y_i(t) = c(a,b)\Lambda^a_b(t) + d(a)\tilde{\Lambda}^a_a(t)$$

This means that the input measurement process comprises Bosonic noise plus the counting components (zero grading) of the Fermionic supersymmetric noise. We cannot measure the noncounting (unity grading) Fermionic components $\xi^a_b(t), \sigma(a,b) = 1$ as they would not lead to non-demolition noise.

Note that $\tilde{\Lambda}^a_a(t)$ commutes with $d\Lambda^c_d(T), d\tilde{\xi}^c_d(T)$ for $T \geq t$, the former being obvious and the latter being because $\tilde{\Lambda}^a_a(t)$ commutes with $d\tilde{\Lambda}^c_d(T)$ and with $\tilde{L}ambda^c_c(T)$ for all a, c, d. Of course, $\Lambda^a_b(t)$ also commutes with $d\Lambda^c_d(T), d\tilde{\xi}^c_d(T)$ for $T \geq t$. In particular, it follows that $Y_i(t)$ commutes with $d\Lambda^c_d(T), d\tilde{\xi}^c_d(T)$ for all $T \geq t$. Thus a candidate for output non-demolition noise is given by

$$Y_o(t) = U(t)^* Y_i(t) U(t)$$

Note that if $a \neq b$, $\tilde{\Lambda}^a_b(t)$ would not commute with $\tilde{\Lambda}^c_c(T)$ for $T \geq t$. It follows therefore that we cannot have terms $\tilde{\Lambda}^a_b(t)$ with $a \neq b$ in $Y_i(t)$ if $Y_o(t) = U(t)^* Y_i(t) U(t)$ is to be non-demolition. Note that all these facts are based on the fact that for the non-demolition property of Y_o, we require that $U(T)^* Y_i(t) U(T) = Y_o(t), T \geq t$ or equivalently that $d_T(U(T)^* Y_i(t) U(T)) = 0, T \geq t$ and the conditions for unitarity of $U(T)$, ie, of $d_T(U(T)^* U(T)) = 0$. We now also assume that the system operators L^a_b, M^a_b are functions of a classical Markovian parameter process $\phi(t)$ having infinitesimal generator K. We also make noisy measurements on this classical parameter, ie, these measurements are

$$dZ(t) = h(\phi(t))dt + dV(t)$$

where $V(.)$ is a classical Levy process. We shall consider the general case when $V(.)$ is correlated with $\phi(.)$ in such a way that the conditional distribution of$(dV(t), d\phi(t))$ given $(V(s), \phi(s), s \leq t)$ is a function of only $\phi(t)$. An example of this is as follows: Suppose

$$d\phi(t) = F_1(t, \phi(t))dt + F_2(t, \phi(t))d\tilde{V}(t) + F_3(t, \phi(t))dV(t)$$

where $\tilde{V}(.)$ is another Levy process independent of $V(.)$. Theh, ϕ is a Markov process and the distribution of $(d\phi(t), dV(t))$ given $\phi(s), V(s), s \leq t$ is clearly a function of only $\phi(t)$ because $(dV(t), d\tilde{V}(t))$ is independent of $(\phi(s), V(s), s \leq t)$. The latter is because $(dV(t), d\tilde{V}(t))$ is independent of $(V(s), \tilde{V}(s)), s \leq t$. Note that $\phi(t)$ can be expressed as a function of $(V(s), \tilde{V}(s)), s \leq t$ and $\phi(0)$ and it is being implicitly assumed that $\phi(0)$ is independent of the processes $V(s), \tilde{V}(s), s \geq 0$. Note further that for this model,

$$d\phi(t) \otimes dV(t)^{\otimes k} = (F_3(t, \phi(t)) \otimes I) dV(t)^{\otimes k+1}, k \geq 1$$

which is nonzero.

The non-demolition measurement algebra upto time t is thus given by

$$\eta_o(t) = \sigma(Y_o(s), Z(s) : s \leq t)$$

A Heisenberg observable at time t is a linear combination of terms of the form

$$j_t(f \otimes X) = j_t(fX) = f(\theta(t)) j_t(X)$$

where

$$j_t(X) = U(t)^* X U(t)$$

with X being a system observable. Its conditional expectation given the output measurements upto time t is

$$\pi_{0t}(fX) = \mathbb{E}(j_t(fX) | \eta_o(t)) - - - (1)$$

In deriving the filter, we shall also require

$$\pi_{1,t}(fX) = \mathbb{E}(j_t(fX.\tilde{G}(t)) | \eta_o(t)) - - - (2)$$

where

$$\tilde{G}(t) = (-1)^{\tilde{\Lambda}(t)}$$

Note that $\tilde{G}(t)$ commutes with $Y_i(s), s \leq t$ and also with $\tilde{\Lambda}_c^c(T)$. Of course, $\tilde{G}(t)$ also commutes with $d\Lambda_b^a(T), d\tilde{\Lambda}_b^a(T)$ for $T \geq t$. It follows therefore that

$$Y_o(t) = j_t(Y_i(t)) = j_T(Y_i(t)), T \geq t$$

and hence from the commutativity of $f, X, \tilde{G}(t)$ with $Y_i(s), s \leq t$ that $j_t(f.X.\tilde{G}(t))$ commutes with with $Y_o(s) = j_t(Y_i(s)), s \leq t$. Therefore, the conditional expectation (2) is well defined. Note that

$$\pi_{m,t}(fX) = \mathbb{E}(j_t(fX.\tilde{G}(t)^m) | \eta_o(t)), m = 0, 1 mod 2$$

We can describe the optimal filter by specifying the coefficients in the classical sde

$$d\pi_{m,t}(fX) = F_{m,t}(fX)dt + \sum_{k \geq 1} G_{1,m,k,t}(fX) dY_o(t)^k + \sum_{k \geq 1} G_{2,m,k,t}(fX) dZ(t)^{\otimes k}, m = 0, 1 - - - (3)$$

These coefficients $F_{m,t}(X), G_{k,m,t}(fX), k = 1,2, m \geq 1$ are measurable w.r.t the algebra $\eta_o(t)$ which is an Abelian algebra. That is why we refer to (3) as a classical sde also called a stochastic Schrodinger equation. Note also the absence of terms like $dY_o(t)^k dZ(t)^{\otimes m}$ for $k, m \geq 1$ because these terms all vanish. We observe that

$$dY_o(t) = dj_t(Y_i(t)) = dY_i(t) + dU(t)^* Y_i(t)U(t) + U(t)^* Y_i(t)dU(t) + dU(t)^* Y_i(t)dU(t)$$

$$+dU(t)^* dY_i(t)U(t) + U(t)^* dY_i(t)dU(t)$$

$$= dY_i(t) + dU(t)^* dY_i(t)U(t) + U(t)^* dY_i(t)dU(t) - - - (4)$$

owing to the fact that $U(t)$ is unitary and that the conditions for the unitarity of $U(t)$ namely, $d(U(t)^*U(t)) = 0$ on the system operators L^a_b, M^a_b do not interfere with the presence of $Y_i(t)$ in the centre owing to the fact that the noise differentials in $dU(t)$ and $dU(t)^*$ are of the form $d\Lambda^a_b(t)$ and $\tilde{G}(t)^{\sigma(a,b)} d\tilde{\Lambda}^a_b(t)$, both of which commute with $Y_i(t)$ primarily because $\tilde{G}(t)$ commutes with $\tilde{\Lambda}^c_c(t)$. In view of the formula (4) and the quantum Ito formula

$$d\tilde{\xi}^a_b(t).d\tilde{\Lambda}^c_c(t) = \tilde{G}(t)^{\sigma(a,b)} d\tilde{\Lambda}^a_b(t) d\tilde{\Lambda}^c_c(t)$$

$$= \tilde{G}(t)^{\sigma(a,b)} \epsilon^a_c d\tilde{\Lambda}^c_b(t)$$

$$= \epsilon^a_c \tilde{G}(t)^{\sigma(b,c)} d\tilde{\Lambda}^c_b(t) = \epsilon^a_c d\tilde{\xi}^c_b(t)$$

it follows that we can write

$$dY_o(t) = j_t(Q(a,b,1)) d\Lambda^a_b(t) + j_t(Q(a,b,2))\tilde{G}(t)^{\sigma(a,b)} d\tilde{\Lambda}^a_b(t) - - - (5)$$

for some system operators $Q(a,b,k), k = 1,2$. It follows that we can write

$$dY_o(t)^k = j_t(Q(a,b,1,k)) d\Lambda^a_b(t) + j_t(Q(a,b,2,k)\tilde{G}(t)^{\sigma(a,b)}) d\tilde{\Lambda}^a_b(t) - - - (6)$$

where by induction,

$$j_t(Q(a,b,1,k+1)) d\Lambda^a_b(t) + j_t(Q(a,b,2,k+1)\tilde{G}(t)^{\sigma(a,b)}) d\tilde{\Lambda}^a_b(t)$$

$$= dY_o(t).dY_o(t)^k =$$

$$[j_t(Q(c,d,1)) d\Lambda^c_d(t) + j_t(Q(c,d,2)\tilde{G}(t)^{\sigma(c,d)}) d\tilde{\Lambda}^c_d(t)] \times [j_t(Q(a,b,1,k)) d\Lambda^a_b(t) + j_t(Q(a,b,2,k)\tilde{G}(t)^{\sigma(a,b)}) d\tilde{\Lambda}^a_b(t])$$

$$= j_t(Q(c,d,1)Q(a,b,1,k))\epsilon^c_b d\Lambda^a_d(t) + j_t(Q(c,d,2)Q(a,b,2,k)\tilde{G}(t)^{\sigma(c,d)+\sigma(a,b)})\epsilon^c_b d\tilde{\Lambda}^a_d(t)$$

$$= j_t(Q(c,d,1)Q(a,b,1,k))\epsilon^c_b d\Lambda^a_d(t) + j_t(Q(c,d,2)Q(a,b,2,k)\tilde{G}(t)^{\sigma(a,d)}\epsilon^c_b d\tilde{\Lambda}^a_d(t)$$

(Note that

$$d\Lambda^c_d(t) d\Lambda^a_b(t) = \epsilon^c_b d\Lambda^a_d(t),$$

$$d\tilde{\Lambda}^c_d(t) d\tilde{\Lambda}^a_b(t) = \epsilon^c_b d\tilde{\Lambda}^a_d(t),$$

$$d\Lambda^c_d(t) d\tilde{\Lambda}^a_b(t) = 0$$

which gives on comparing coefficients,

$$Q(a,d,1,k+1) = Q(c,d,1)Q(a,b,1,k)\epsilon^c_b,$$

$$Q(a,d,2,k+1) = Q(c,d,2)Q(a,b,2,k)\epsilon^c_b$$

or equivalently, in compact notation,

$$Q(1,k{+}1) = Q(1)\epsilon Q(1,k), Q(2,k{+}1) = Q(2)Q(2,k), k \geq 1, Q(1,1) = Q(1), Q(2,1) = Q(2)$$

Now we can as discussed above, assume that

$$\mathbb{E}(df(\theta(t)).dV(t)^{\otimes k}|\eta_o(t)) = K(k)f(\theta(t))dt, k \geq 0$$

and in particular,

$$\mathbb{E}(df(\theta(t))|\eta_o(t)) = K(0)f(\theta(t))dt$$

where $K(0)$ is the generator of the Markov process $\theta(.)$. $K(k)$ is a linear operator acting in an appropriate space of functions f defined on the Markov parameter state space. Now compute for a system observable X,

$$dj_t(X) = j_t(\theta(a,b,0,X))d\Lambda^a_b(t) + j_t(\theta(a,b,1,X)\tilde{G}(t)^{\sigma(a,b)})d\tilde{\Lambda}^a_b(t)$$

$$dj_t(X.\tilde{G}(t)) = j_t(\theta(a,b,2,X)\tilde{G}(t))d\Lambda^a_b(t) + j_t(\theta(a,b,3,X)\tilde{G}(t)^{\sigma(a,b)+1})d\tilde{\Lambda}^a_b(t)$$

Both of these equations can be summarized within a single compact notation as

$$dj_t(X.\tilde{G}(t)^m) = j_t(\theta(a,b,m,0,X)\tilde{G}(t)^m)d\Lambda^a_b(t)$$

$$+j_t(\theta(a,b,m,1,X)\tilde{G}(t)^{\sigma(a,b)+m})d\tilde{\Lambda}^a_b(t), m = 0,1$$

Therefore, for $m = 0,1$,

$$dj_t(fX.\tilde{G}(t)^m) = df(\theta(t))j_t(X.\tilde{G}(t)^m) + f(\theta(t))dj_t(X.\tilde{G}(t)^m)$$

$$= df(\theta(t))j_t(X.\tilde{G}(t)^m)) + f(\theta(t))j_t(\theta(a,b,m,0,X)\tilde{G}(t)^m)d\Lambda^a_b(t)$$

$$+f(\theta(t))j_t(\theta(a,b,m,1,X)\tilde{G}(t)^{\sigma(a,b)+m})d\tilde{\Lambda}^a_b(t)$$

and therefore,

$$\mathbb{E}[dj_t(fX.\tilde{G}(t)^m)|\eta_o(t)] =$$

$$\pi_{m,t}((K(0)f).X)dt + \pi_{m,t}(f.\theta(a,b,m,0,X))u_a(t)\bar{u}_b(t)dt$$

$$+\pi_{m+\sigma(a,b)}(f.\theta(a,b,m,1,X))v_a(t)\bar{v}_b(t)dt - - - (7)$$

where we are assuming the bath to be in the coherent state

$$|\phi(u{\oplus}v) >= exp(-|u{\oplus}v|^2/2)|e(u{\oplus}v) >= exp(-|u|^2/2).exp(-|v|^2/2)|e(u) > \otimes|e(v) >$$

$$= |\phi(u) > \otimes|\phi(v) >= |\phi(u) \otimes \phi(v) >$$

Note that the term $df(\theta(t))dj_t(X.\tilde{G}(t)^m) = 0$ because $df(\theta(t))d\Lambda^a_b(t) = df(\theta(t))d\tilde{\Lambda}^a_b(t) = 0$. Therefore, for $k \geq 1$,

$$\mathbb{E}[dj_t(fX.\tilde{G}(t)^m).dY_o(t)^k|\eta_o(t)] =$$

$$\mathbb{E}[[df(\theta(t))j_t(X.\tilde{G}(t)^m)) + f(\theta(t))j_t(\theta(a,b,m,0,X)\tilde{G}(t)^m)d\Lambda^a_b(t)$$

$$+f(\theta(t))j_t(\theta(a,b,m,1,X)\tilde{G}(t)^{\sigma(a,b)+m})d\tilde{\Lambda}_b^a(t)].$$
$$\times[j_t(Q(c,d,1,k))d\Lambda_d^c(t)+j_t(Q(c,d,2,k)\tilde{G}(t)^{\sigma(c,d)})d\tilde{\Lambda}_d^c(t)]|\eta_o(t)]$$
$$=\pi_{m,t}((K(0)f)X.Q(c,d,1,k))u_c(t)\bar{u}_d(t)dt$$
$$+\pi_{m+\sigma(c,d),t}((K(0)f)Q(c,d,2,k))v_c(t)\bar{v}_d(t)dt$$
$$+\pi_{m,t}(f.\theta(a,b,m,0,X)Q(c,d,1,k))\epsilon_d^a u_c(t)\bar{u}_b(t)dt$$
$$+\pi_{m+\sigma(a,b)+\sigma(c,d),t}(f.\theta(a,b,m,1,X)Q(c,d,2,k))\epsilon_d^a v_c(t)\bar{v}_b(t)dt$$
$$=\pi_{m,t}((K(0)f)X.Q(c,d,1,k))u_c(t)\bar{u}_d(t)dt$$
$$+\pi_{m+\sigma(c,d),t}((K(0)f)Q(c,d,2,k))v_c(t)\bar{v}_d(t)dt$$
$$+\pi_{m,t}(f.\theta(a,b,m,0,X)Q(c,d,1,k))\epsilon_d^a u_c(t)\bar{u}_b(t)dt$$
$$+\pi_{m+\sigma(b,c),t}(f.\theta(a,b,m,1,X)Q(c,d,2,k))\epsilon_d^a v_c(t)\bar{v}_b(t)dt---(8)$$

We also require for $k\geq 1$,

$$\mathbb{E}[dj_t(fX.\tilde{G}(t)^m).dZ(t)^{\otimes k}|\eta_o(t)]=$$
$$\mathbb{E}[[df(\theta(t))j_t(X.\tilde{G}(t)^m))+f(\theta(t))j_t(\theta(a,b,m,0,X)\tilde{G}(t)^m)d\Lambda_b^a(t)$$
$$+f(\theta(t))j_t(\theta(a,b,m,1,X)\tilde{G}(t)^{\sigma(a,b)+m})d\tilde{\Lambda}_b^a(t)].$$
$$\times dV(t)^{\otimes k}|\eta_o(t)]$$
$$=\pi_{m,t}((K(k)f)X)dt---(9)$$

We are now in a position to complete the formulation of the quantum-Boson-Fermion Belavkin filter for the case when Fermionic noise is obtained by Poisson twisting of Bosonic noise: The basic orthogonality principle gives

$$\mathbb{E}[(j_t(f.X.\tilde{G}(t)^m)-\pi_{m,t}(f.X))C(t)|\eta_o(t)]=0,m=0,1---(10)$$

where

$$dC(t)=\sum_{k\geq 1}(f_k(t)dY_o(t)^k+g_k(t)^T dZ(t)^{\otimes k})C(t),t\geq 0,C(0)=1---(11)$$

with $f_k(t)$ being any real valued function of time and $g_k(t)$ any vector valued function of time of the same dimension as $Z(t)^{\otimes k}$. Taking the differential of (10) and making use of the quantum and classical Ito formulas and the arbitrariness of the functions f_k, g_k gives us

$$\mathbb{E}[dj_t(f.X.\tilde{G}(t)^m)-d\pi_{m,t}(f.X))|\eta_o(t)]=0,m=0,1---(12a),$$

$$\mathbb{E}[(j_t(f.X.\tilde{G}(t)^m)-\pi_{m,t}(f.X))dY_o(t)^k|\eta_o(t)]$$
$$+\mathbb{E}[(dj_t(f.X.\tilde{G}(t)^m)-d\pi_{m,t}(f.X))dY_o(t)^k|\eta_o(t)]=0,m=0,1,k\geq 1---(12b),$$
$$\mathbb{E}[(j_t(f.X.\tilde{G}(t)^m)-\pi_{m,t}(f.X))dZ(t)^{\otimes k}|\eta_o(t)]$$

$$+\mathbb{E}[(dj_t(f.X.\tilde{G}(t)^m)-d\pi_{m,t}(f.X))dZ(t)^{\otimes k}|\eta_o(t)]=0, m=0,1,k\geq 1---(12c)$$

These equations result in an infinite sequence of linear algebraic equations for the filter coefficients $G_{1,m,k,t}(fX), G_{2,m,k,t}(fX), m=0,1,k\geq 1$ with f varying over the algebra of bounded Borel functions of the parameter θ and X varying over the linear space of self-adjoint operators in the system Hilbert space. We leave the completion of the calculations as an exercise to the interested reader.

Remark: In the quantization of the gravitational field interacting with other quantum fields, after spatial discretization, the problem is described by a constrained Hamiltonian operator of all the position and momentum operators obtained by discretizing the corresponding position and momentum fields w.r.t the three spatial variables. The Lie brackets then appearing in noisy Heisenberg dynamics are then to be replaced with Dirac brackets and we can in principle then obtain a real time filter for estimating the joint state of the gravitons and all the other particles associated with the different fields interacting with gravity. From this estimated state, we can using the partial trace operation over the other field component Hilbert spaces, obtain a time varying estimate of the graviton state alone based on non-demolition measurements of the output noise component fields associated with the other particles. In other words, for example, by measuring output continuous and discrete counting electromagnetic photon noise or output electron-positron counting noise or output continuous and discrete counting non-Abelian gauge field noise, hope to obtain real time estimates of the graviton state and hence construct fine detectors of quantum mechanical effects of gravitational waves. It should be remarked, that the HP system dynamics affects the measurement noise (which is the reason why we are able to extract information about the system from the scattered noise) but the non-demolition property of the measurement noise implies that it does not affect the future system dynamics (which is why we obtain reliable estimates about the system state from measurements of the output/scattered noise).

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