Review of: "Riemann Hypothesis on Grönwall's Function"

Yannick Saouter[1]

1 IMT Atlantique

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This article tries to give a new criterion for the Riemann hypothesis based on the value of the Gronwall function for collossally abundant numbers.

However I am not convinced by the exposure of arguments.

First, you suppose that there is not an infinity of pair of consecutive CA numbers N and N' such that $G(N')\geq G(N)$. You claim then that there is a unique CA number N'' which maximizes function G. If so, there is no need to recourse to anything else: by definition, N'' is then the largest XA number and the Riemann hypothesis is false. However, I do not see by your arguments why N'' should be unique. It could be, at least from the common logic, that all CA numbers larger than N'' meet the same value. Luckily, if you refer to the definition of Nazardyonavi and Yabukovich of XA numbers (def 3), in fact none of these numbers is a proper XA number since the inequality is strict. Therefore again, there is not an infinity of XA numbers and the Riemann Hypothesis is false.

In the second part of the proof, you define N'' as the largest XA number less than N' where N,N' is a pair of consecutive CA numbers. You state then that N'' is a CA number by prop 3. I do not see how prop 3 can be used to obtain that result. Then you state that if $N<n<N'$, then $G(N)\leq G(n)\leq G(N')$ and this is a flaw. Prop 1 states only that $G(n)\leq G(N)$ and $G(n)\leq G(N')$. The Gronwall function is not monotonic. For instance, G(2^n) is less than 2/log(n) and thus tends to 0 when n goes to infinity. However, from Gronwall theorem, at the infinity, G assumes values arbitrary close to exp(gamma). Luckily again, prop 3 requires only than $N<n<N'$ and in this case if n is XA, N' is also XA. The inequation $G(N)\leq G(n)\leq G(N')$ is then a simple consequence of the definition of XA numbers. Then you consider the set S of CA integers M such that $N''<M<N'$. Note that this set might be empty and therefore M'' would be undefined. If M'' exists and is XA, you obtain a possible new large XA number. However, you did not prove that if N' goes to infinity, so does M''. Moreover, if for instance M''=N, then M'' goes to infinity but since you do not have exhibited a XA number n with $N<n<N'$, you cannot conclude that N' is XA.

From the preceding, although the claimed theorem could be correct, the presented material is not convincing. I have also the belief that this theorem is a simple consequence of the properties of Gronwall function and Robin's criterion.