Cryptocurrency market risk analysis: evidence from FZL function

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Abstract

Cryptocurrencies are risky currencies due to their extreme price volatilities and requires an estimation of coherent risk measures for an effective portfolio optimization and risk management. We focus on seven cryptocurrencies (Bitcoin, Ethereum, Litecoin, Ripple, Das, Monero, and Steller) and provide empirical application of Fissler and Ziegel joint loss dynamic models (FZL) for joint Value-at-Risk (VaR) and Expected Shortfall (ES) in a cryptocurrency context at α= 0.01 and α= 0.025 risk levels. Results show Ethereum and Steller as less risky currencies followed by Monero, Das, Litecoin, Bitcoin, and largest for Ripple suggesting that Ethereum and Steller requires the least capital to absorb losses. Following this result, we argue that market participants interested in cryptocurrencies can follow the rankings in this study to hedge, calculate margins, and capital requirement to maximize utility whiles minimizing risk to ensure financial stability in the global economy.

Keywords: Cryptocurrencies; FZL; Value-at-Risk; Expected Shortfall; GAS framework.

JEL classification: C32, F36, F37, G10, G15.

1. Introduction

Following Basel III international regulatory framework on the development of enhanced risk management system for financial institutions after the global financial crises in 2008/2009, the international financial system has faced a new challenge with the introduction of cryptocurrencies[1], decentralized virtual currencies with excess price volatility (Caporale and Zekokh 2019). Since then, studies have modelled volatility in cryptocurrency market but not incorporating volatility during extreme tail events by applying Fissler and Ziegel joint loss dynamic models (FZL) for joint Value-at-Risk (VaR) and Expected Shortfall (ES) in cryptocurrency market. The studies mainly focused on the predictable variance and missed out on the tails of distribution of the cryptocurrencies (Bouri, Mahamitra, Gupta, and Roubaud, 2018; Bouri, Shahzad, and Roubaud, 2019; Corbet et al., 2018; Omane-Adjepong, Alagidede, and Akosah, 2019; Catania et al., 2018; Katsiampa, 2019; Yi et al., 2018; Koutmos, 2018; Ji et al., 2018). Notably, extreme market conditions largely reflect in the distributional properties of returns referred to as tail events. Tail events particularly have consequences for risk management and portfolio diversification.

Other approaches of examining volatility in cryptocurrency market using VAR, GARCH, and Copula methodology are also evident in the literature (Gkillas, Bekiros, and Siriopoulos, 2018; Gkillas and Katsiampa, 2018; Trabelsi, 2018; Borri, 2019; Huynh et al., 2018; Huynh, 2019). Evidence from such studies suggests that cryptocurrencies are highly dependent at the tails and are exposed to tail-risk. The empirical literature also has evidence studies that have attempted to estimate and predict Value-at-Risk (VaR) and Expected Shortfall (ES) in the cryptocurrency market. For instances, Caporale and Zekokh (2018) model the volatility of Bitcoin, Litecoin, Ethereum, and Ripple applying GARCH models to a one-step ahead forecast of Expected Shortfall and Value-at-Risk. The results show that GARCH models incorrectly predict ES and VaR which can lead to ineffective portfolio optimization, risk management and derivative pricing. The authors recommend asymmetric distributions for such analysis.

The study of Troster, Tiware, Shahbaz, and Macedo (2018) model Bitcoin return and risk using GAS and GARCH framework. Findings show that Generalized Autoregressive Score (GAS) framework with heavy tail asymmetric distributions provided best goodness-of-fit and out-of-sample forecast compared to heavy-tailed GARCH models. Angelini and Emili (2018) predict the volatility of six cryptocurrencies using GARCH-M, EGARCH, APARCH, TGARCH, and the simple GARCH for 700 daily cryptocurrency prices in a one-step ahead forecast in a recursive manner and findings show that the EGARCH performed better than the other GARCH models. Konstantinos and Katsiampa (2018) make use of extreme value technique to estimate ES and VaR of five cryptocurrencies: Bitcoin, Bitcoin Cash, Ethereum, Ripple and Litecoin. The results of the study show Bitcoin cash as the riskiest cryptocurrency whiles Litecoin and Bitcoin have the least risk profile.

Nevertheless, all the above studies missed out on the FZL function for joint VaR and ES in a univariate GAS framework. As noted by Catania et al. (2018), the time series of cryptocurrencies exhibit the stylized facts of other financial time series which includes extreme observations, time-varying volatility, and asymmetric volatility process even though they provide their owners immense profit when invested at the right time. These stylized facts have been confirmed by Corbet et al. (2018a), Troster, Tiware, Shahbaz, and Macedo (2018), Borri (2019), and Huynh (2019). The authors note that, the newly emerged digital assets comprises USD billions everyday but exhibit extreme price volatility, are fat-tailed, and exposed to tail risk and requires coherent risk measures such as Value-at-Risk and Expected Shortfall in a model which can capture all conventional moments without assuming normality.

Value-at-Risk measures a quantile of the distribution of an asset and ignores critical information concerning the tails of the distributions beyond this quantile, whiles Expected Shortfall measure the average return of an asset conditional on the return being below its Value-at-Risk level (Aas and Haff, 2006). Expected Shortfall which robustly measure tail risk was proposed by Basel III[2] after the global financial crises to complement and in part substitute Value-at-Risk measure which do not comprehensively capture tail risk (BIS, 2013; Basel Committee, 2010; Patton, Ziegel, and Chen, 2019). Basically, Expected Shortfall is Value-at-Risk calibrated to stress market conditions with regulatory capital sufficient under both tranquil and extreme market turmoil (BIS, 2013). However, despite being a coherent risk measure (Artzner et al., 1999) in contrast with VaR, ES is not elicitable[3] and robust in estimation procedures as in VaR (Fissler & Ziegel, 2016; Cont, Deguest, & Scandolo, 2010; Burzoni, Peri, & Ruffo, 2017; Nolde & Ziegel, 2017; Fissler, Ziegel, & Gneiting, 2015).

Nonetheless, the study of Fissler and Ziegel (2016) show that the couple ES and VaR is jointly elicitable at higher order. Subsequently, the authors proposed a joint loss function for ES and VaR hereafter referred to as FZL which replaces the traditional backtesting in Basel III with comparative backtesting. Given that ES and VaR are coherent risk measure, there is no empirical application of joint loss dynamic models (FZL) for joint VaR and ES in cryptocurrency markets risk analysis which is the major contribution of this study. Given the extreme price volatility of cryptocurrencies coupled with their exposure to tail risk, it is important to estimate appropriate risk metrics by choosing suitable asymmetric distributions that can adequately capture tail risk, volatility clusters, and skewness which can be used for calculating margins, capital requirement, and hedging to ensure financial stability in the global economy.

As required by Basel III that financial institutions worldwide are to implement and regularly disclose VaR and ES of required capital, VaR and ES will gain increased attention form bank supervisors, risk managers, regulators, and investors worldwide. Thus, to account for the empirical limitations of ignoring the stylized facts of cryptocurrency returns in a coherent risk measure, we apply six different asymmetric distributions in the univariate GAS framework proposed by Harvey (2013) and Creal et al. (2013) to estimate and forecast VaR and ES using FZL function for joint VaR and ES. This has implications for investors and risk managers, especially financial institution risk management[4] on the occasion that they invest in cryptocurrencies. The intuition is drawn from Extreme Value Theory (EVT)[5] by Fisher and Tippett (1928). The theory concerns itself with the modelling of the tails of a distribution and its key results. The EVT literature has become popular because it recognizes extreme observations as an important stylised fact for the estimation of risk measures. As Cecchinato (2010) notes it is important to know the stylized facts of financial data in order to choose the best fitting model for it. Ignoring heavy tails, time-varying volatility, asymmetric response to bad and good news, and skewness can lead to underestimating risk with a possible consequence of default of a firm, bank, or an investor which can impair financial stability. Results from FZL function for joint VaR and ES show that, at both $\alpha = 0.01$ and $\alpha = 0.025$ risk levels, Ethereum and Steller has the least risk profile followed by Monero, Das, Litecoin, Bitcoin, and largest for Ripple. Our results suggest that at $\alpha = 0.01$ and $\alpha = 0.025$ risk levels, Ethereum and Steller requires the least capital to absorb losses followed by Monero, Das, Litecoin, Bitcoin, and Ripple.

The remainders of the study are structured as follows. Section 2 covers a description of the methodology. Section 3 covers a description of data and statistical properties. Section 4 captures the results and discussion on FZL function. Section 5 is the conclusion and policy implications.

2. Methodology

The section describes the approach used to apply the FZL function for joint VaR and ES in cryptocurrency market risk analysis. Our approach is based on the univariate GAS framework. Since FZL and GAS are score driven, FZL can better be estimated by GAS model given the fundamental property of score function between them.

2.1 Univariate GAS model specification

The univariate GAS framework also known as Score Driven (SD) models and Dynamic Conditional Score models proposed by Harvey (2013) and Creal et al. (2013) is a valuable tool for prediction and signal extractions which is more robust to volatility clusters, skewed, and fat-tailed distributions (Ardia, Boudt, and Catania, 2018; Troster, Tiware, Shahbaz, and Macedo, 2018).

GAS models introduce a driving mechanism for time-varying parameters by using the score function of the predictive model density at time t instead of only higher moments and means and can model all types of time series data. GAS models encompass popular models such as the autoregressive conditional duration, generalized autoregressive conditional heteroskedasticity, autoregressive conditional intensity, poisson count models with time-varying mean, and the dynamic copula models (Creal, Koopman, and Lucas, 2012). Empirical application of GAS model for financial risk forecasting include; Oh and Patton (2016) for systematic risk, Blasques, Koopman, Lucas, and Schaumburg (2016b) for spatial econometrics, Harvey and Thiele (2016) for dependence modeling, and Harvey and Sucarrat (2014) for market risk.

The study follows the GAS framework proposed by Harvey (2013) and Creal et al. (2013), which is specified as:

$$r_t \left| F_{t-1} \sim p\left(r_t; \theta_t\right) \right. \qquad (1)$$

Where F_{t-1} denotes the past values of cryptocurrency returns (r_t) up to $t-1$, $\theta_t \in \Theta \subseteq R^J$ represents a time-varying parameters' vector that fully identifies $p(.)$, and $p\left(r_t; \theta_t\right)$ is the returns conditional distribution.

The GAS model in equation (1) is defined with autoregressive component incorporated as:

$$\theta_{t+1} = \omega + As_t + B\theta_t, \qquad (2)$$

$$s_t = S_t\left(\theta_t\right)^{\frac{\partial \log p\left(r_t; \theta_t\right)}{\partial \theta_t}}, \qquad (3)$$

Where ω is a vector of constant, A , and B are coefficient matrices, s_t is the steps of the scaled-score vector, $\frac{\partial \log p\left(r_t; \theta_t\right)}{\partial \theta_t}$ is the score of (1) that is appraised at θ_t, and $S_t\left(\theta_t\right)$ [6] is a positive definite scaling matrix that adjusts the shape of the score specified as:

$$S_t\left(\theta_t\right) = E_{t-1}\left[\frac{\partial \log p\left(r_t; \theta_t\right)}{\partial \theta_t} \frac{\partial \log p\left(r_t; \theta_t\right)'}{\partial \theta_t}\right]^{-1} \qquad (4)$$

Where E_{t-1} denotes an expectation with respect to $p\left(r_t; \theta_t\right)$.

Equations 1 to 4 define the GAS framework and we use $GAS(1,1)$ in all estimations in line with Mensah and Alagidede (2017), Ardia, Boudt, and Catania (2016; 2018), and Creal et al. (2013).

2.2 Selected distributions for the univariate GAS model

In the univariate GAS framework; specifications for skewed-Gaussian (SNORM), student-t (STD), skewed-student-t (SSTD) (Fernández and Steel, 1998), asymmetric student-t with two tail decay parameters (AST), asymmetric student-t with one tail decay parameter (AST1) (Zhu and Galbraith, 2010; 2011), and asymmetric Laplace (ALD) (Kotz et al., 2012) are considered[7]. These distributions are selected because of their flexibility to incorporate the features of financial returns which are skewed, fat-tailed with volatility clustering (McNeil et al., 2015) in forecasting the FZL for cryptocurrencies.

2.2.1. STD, SSTD, and SNORM

A skewness parameter in general controls the asymmetry of the central part of a distribution. In risk management and finance, STD, SNORM, and SSTD has single tail parameter used to model tin/heavy tails and skewness in conditional distribution of financial returns (Zhu and Galbraith, 2010; Fernández and Steel, 1998). The general form of STD, SSTD, and SNORM distributions is defined as:

$$r_t \middle| \mathrm{F}_{t-1}\ SSTD\left(r_t, \mu, \sigma_t, \zeta, v\right) \qquad (5)$$

Where F_{t-1} denotes autoregressive conditional distribution of r_t, $\mu \in \mathrm{R}$ is the location parameter, $\sigma_t > 0$ is time-varying scale, $v > 2$ is shape parameter, and $\zeta > 0$ denotes skewness. In line with Ardia, Boudt, and Catania (2016; 2018), Bauwens and Laurent (2005) we parameterize equation (5) so that $Var_{t-1}\left[r_t\right] = \sigma_t^2$, and $\mathrm{E}_{t-1}\left[r_t\right] = \mu$. We recover as special cases SSTD imposing SNORM when $\zeta = \infty$, and SSTD imposing STD when $\zeta = 1$.

2.2.2. AST, AST1

The AST distributions have skewness and two tail parameters which control the left and right tail behavior of financial returns. The two tail parameters increase the capacity to fit and forecast financial returns in the tail regions which is crucial for risk management (Zhu and Galbraith, 2010). The AST and AST1 distributions in this study have zero location parameter, unity (one) scale parameter with probability density function expressed as:

$$AST\left(r_t;\theta\right) = \begin{cases} \frac{1}{\sigma}\left[1 + \frac{1}{\vartheta_1}\left(\frac{r_t-\mu}{2\alpha\sigma K\left(\vartheta_1\right)}\right)^2\right]^{-\left(\vartheta_1+1\right)/2}, & r_t \le \mu \\ \frac{1}{\sigma}\left[1 + \frac{1}{\vartheta_2}\left(\frac{r_t-\mu}{2(1-\alpha)\sigma K\left(\vartheta_2\right)}\right)^2\right]^{-\left(\vartheta_2+1\right)/2}, & r_t > \mu \end{cases} \qquad (6)$$

where $\theta = (\alpha, \vartheta_1, \vartheta_2, \mu, \sigma)^T$, α is skewness, ϑ_1 is left tail, ϑ_2 is right tail, μ is location, σ is scale parameter, with

$K\vartheta = (\Gamma(\vartheta + 1)\backslash 2)/[\sqrt{\pi\vartheta}\Gamma(\vartheta/2)]$, for $\vartheta_1, \vartheta_2 \in \vartheta$.

2.2.3. ALD

The most suitable skewed simplification of classical Laplace law which robustly measure fat-tail, skewness, and leptokurtic characteristics of financial returns is ALD (Kotz et al., 2012; Zhao et. al., 2015) with probability density function specified as:

$$f\left(r_t; \theta, \sigma, \kappa\right) = \frac{\kappa(1-\kappa)}{\sigma} \exp\left(- \frac{\left(r_t - \theta\right)}{\sigma} \left[\kappa - I\left(r_t \leq \theta\right)\right]\right), \qquad (7)$$

where $-\infty < \theta < \infty$ is location parameter, $\sigma > 0$ is scale parameter, and $0 < \kappa < 1$ is skewness with indication function as $I(.)$.

We employ Kotz et al. (2012), Blasques, Koopman, and Lucas (2014) maximum likelihood estimator jointly with one-step ahead forecast to estimate the parameters in all the distributions.

2.3. FZL Function for joint VaR and ES

The two standard risk measures used in finance are Value-at-Risk (VaR) and Expected Shortfall (ES) (Jorion, 1997; Ardia, Boudt, and Catania, 2018). However, these two standard risk measures fall short in their own rights under the properties of a coherent risk measure (Acerbi and Szekely, 2014; Acerbi and Tasche, 2002). Value-at-Risk is elicitable using the quantile loss function but not sub-additive whereas ES is not elicitable since it has no loss function but coherent and comonotonically additive (Ziegel, 2016; Bellini and Bignozzi, 2015). Nevertheless, Fissler and Ziegel (2016) show that VaR is jointly elicitable with ES, and thus a dynamic model can be built for both. Subsequently, the authors proposed a joint loss function for ES and VaR hereafter referred to as FZL which replaces the traditional backtesting in Basel III with comparative backtesting. Following Fissler and Ziegel (2016), we explore the FZL dynamic model for joint VaR and ES which is based on the univariate GAS framework.

As noted by Artzner et al. (1999) the VaR $\alpha \in (0, 1)$ for a random variable X with a distribution function F is defined as:

$$VaR_\alpha(X) = inf\{x \mid F(x) \leq \alpha\}. \qquad (8)$$

ES is specified as:

$$E\left(X \middle| X > VaR_\alpha\right) \qquad (9)$$

As noted by Fissler and Ziegel (2016) VaR and ES are jointly elicitable as the values of v_t and e_t that minimize the sample average of the loss function:

$$FZ\left(r_t, v_t, e_t, \alpha,\ G_1, G_2\right) \equiv \left(d_t - \alpha\right)\left(G_1\left(v_t\right) - G_1\left(r_t\right) + \frac{1}{\alpha}G_2\left(e_t\right)v_t\right) - G_2\left(e_t\right)\left(\frac{1}{\alpha}d_t r_t - e_t\right) - G_2\left(e_t\right), \qquad (10)$$

where G_1 and G_2 are strictly increasing, $G_2 = 0$, and $G_2^{'} = G_2$. In line with Ardia et al. (2018), Patton et al. (2017), we assume strictly inverse values for VaR and ES by setting $G_1(x) = 0$ and $G_2(x) = \frac{-1}{x}$. We specify the associated FZ loss function for predicting VaR and ES at α risk level at time t as:

$$FZL_t^{\alpha} \equiv \frac{1}{\alpha ES_t^{\alpha}} d_t\left(r_t - VaR_t^{\alpha}\right) + \frac{VaR_t^{\alpha}}{ES_t^{\alpha}} + log\left(-ES_t^{\alpha}\right) - 1, \qquad (11)$$

for $ES_t^{\alpha} \leq VaR_t^{a} \leq 0$. We average FZ losses over the OOS period to calculate the QL.

2.4. FZL backtesting and model comparison

The study follows a typical out-of-sample exercise where model parameters are estimated using in-sample (IS) period of length M, predictions on the conditional distribution in the out-of-sample (OOS) period$H$ and model comparison made according to their out-of-sample performance. Thus, h-step ($h = 1$) ahead daily forecast of the return distribution of cryptocurrencies at time $M + h$ is generated along with corresponding FZL level in a recursive manner until the end of the series T.

We test the null hypothesis of equal predictive accuracy (EPA) of the six asymmetric distributions using the multivariate version (MDM) of Mariano and Preve (2012), and Diebold and Mariano (1995). In line with Blazsek and Hernandez (2018), we rank the six competing models for FZL by calculating the mean absolute error (MAE) $1/T_f \sum_{t=1}^{T_f} |p_t - \bar{p}_t|$ between realized and forecast values and the model with least MAE is chosen as candidate for backtesting.

To verify the precision of predictions and check the correct coverage of unconditional and conditional left-tail of log-returns, four backtesting procedures of FZL forecasts at the risk level[8] $\alpha = 0.01;\ \alpha = 0.25$ of cryptocurrency returns are performed by implementing Kupiec (1995)'s correct unconditional coverage (UC)[9], Christoffersen (1998)'s correct conditional coverage (CC)[10], dynamic quantile (DQ)[11] by Engle and Manganelli (2004), and quantile loss (QL)[12] by (Koenker and Bassett, 1978).

3. Data description and preliminary analysis

We focus on seven cryptocurrencies (Bitcoin, Ethereum, Litecoin, Das, Ripple, Monero, and Steller) for the cryptocurrency market risk analysis. The cryptocurrencies sampled have existed for the past two years and are among the top fifteen currencies by market capitalization and can proxy for the cryptocurrency market. Daily data is sourced from CoinMarketCap and spans 10th August 2015 to 18th February 2019. We calculate returns as change in log price for Monday-to-Friday series.

We split the period of analysis (10/8/2015 – 18/2/2019) into in-sample (IS) (10/8/2015 – 11/12/2017) representing the estimation period and out-of-sample (OOS) (12/12/2017 – 18/2/2019) representing the forecasting period as per out-of-sample forecasting exercise. We choose the OOS as the forecast length since backtesting periods are based on the forecast length. As prescribed by BIS (2013), the OOS is a minimum of one year and percentiles are 99 and 97.5.

Table 1 presents summary statistics on the seven daily cryptocurrency return series studied. In the first panel we present the composite summary statistics on the seven return series. Average daily return ranges from 0.2% for Bitcoin and Ethereum to 0.55% for Steller and daily variance range from 0.18% for Bitcoin and Ethereum to 0.88% for Steller. Litecoin, Ripple, Das, Monero, and Steller are positively skewed except Bitcoin and Ethereum. The second panel shows the in-sample summary statistics with average daily returns of -0.026% for Ripple to 0.084% for Ethereum and daily variance of 0.03% for Das to 0.059% for Ethereum. All the cryptocurrencies are positively skewed except Bitcoin. The out-of-sample summary statistics in the third panel exhibit negative average return for all cryptocurrencies except Litecoin, daily variance range from 0.02% for Bitcoin to 0.099% for Ripple, whiles Bitcoin, Ethereum, Monero depicts negative skewness. In general, kurtosis and skewness values across the board show leptokurtic and non-normality in cryptocurrency returns which is confirmed by the Shapiro-Wilk test by rejecting the normality assumption at all conventional levels of significance. These non normality features can also be observed from the time series plots of the cryptocurrencies in Figure 1 which goes to support the need for using asymmetric distributions in modelling the tail risks of the cryptocurrencies.

Table 1. Summary statistics of Cryptocurrencies

Composite	Bitcoin	Ethereum	Litecoin	Ripple	Das	Monero	Steller
Observ.	885	885	885	885	885	885	885
Mean	0.002	0.002	0.0007	0.0026	0.0021	0.0026	0.0055
Variance	0.0018	0.0018	0.0038	0.0044	0.0035	0.0044	0.0088
Skewness	-0.2356	-0.2356	1.2575	4.5132	0.2983	4.5132	2.1108
Kurtosis	4.7234	4.7234	12.1915	76.4051	3.383	76.4051	17.9168
Normtest.W	0.914	0.914	0.8425	0.5981	0.9463	0.5981	0.8568
Normtest.p	0	0	0	0	0	0	0
In-sample							
Observ.	590	590	590	590	590	590	590
Mean	0.0055	0.0084	0.0042	-0.0026	0.007	0.0047	0.0123
Variance	0.0016	0.0059	0.0037	0.0016	0.003	0.0052	0.0104
Skewness	-0.1033	0.7087	1.3649	0.8047	0.3981	0.9364	2.2475
Kurtosis	6.8739	4.0117	15.9088	6.2963	3.7041	6.2583	17.7008
Normtest.W	0.8752	0.9254	0.7784	0.8876	0.9373	0.9128	0.8447
Normtest.p	0	0	0	0	0	0	0
Out-of-sample							
Observ.	295	295	295	295	295	295	295
Mean	-0.0048	-0.006	-0.0062	0.0128	-0.0078	-0.006	-0.008
Variance	0.002	0.0037	0.0041	0.0099	0.0043	0.0046	0.0053
Skewness	-0.3551	-0.0008	1.1381	3.6817	0.3133	-0.1485	0.6361
Kurtosis	1.7404	1.6286	6.5447	42.5311	2.8943	1.196	5.5911
Normtest.W	0.9616	0.9661	0.9217	0.553	0.9556	0.9813	0.9351
Normtest.p	0	0	0	0	0	0	0

Period of analysis: 10/8/2015 – 18/2/2019 (Out-of -sample: 12/12/2017 – 18/2/2019, In-sample: 10/8/2015 – 11/12/2017). Observ. – observations, Shapiro-Wilk test rejects the normality assumption at all conventional levels of significance.

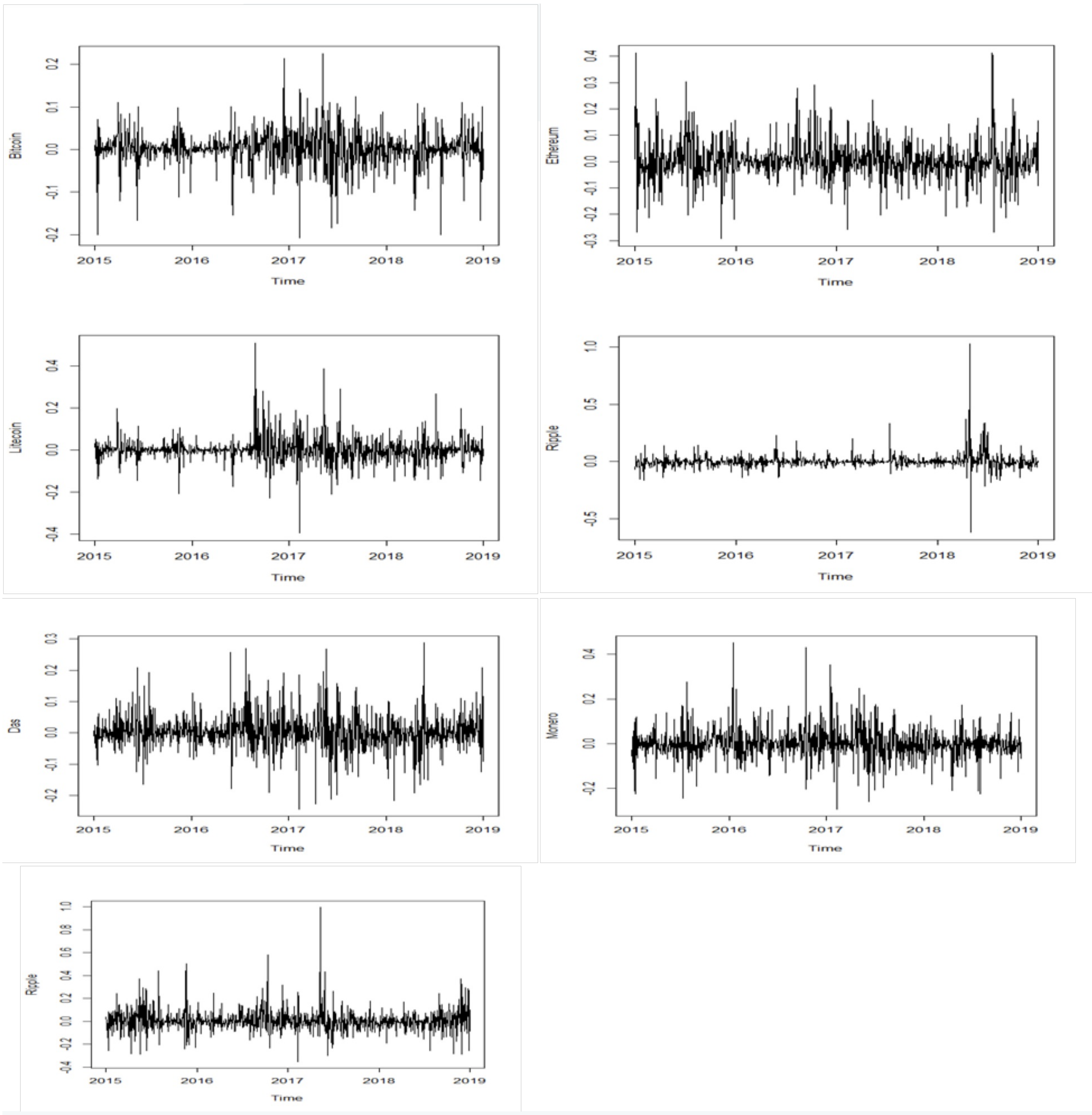

Figure 1. Time series plot of selected Cryptocurrencies

4. Results and Discussion

4.1. Forecasting univariate GAS FZL functions

We apply six different distributional assumptions (SNORM, STD, SSTD, AST, AST1, and ALD) of the univariate GAS models to one-step ahead FZL in the OOS period at $\alpha = 0.01$ and $\alpha = 0.025$ levels. We then apply MDM to the competing models to test the null hypothesis of equal predictive adequacy (EPA) of the models. The MDM test of EPA in Table 2 fails to reject the null hypothesis of EPA at all conventional levels of significance for all cryptocurrencies except Litecoin at 1% level. This outcome raises concerns about the robustness of the test since adjudging six models of different distributional assumptions as being of equal predictive accuracy is almost hard to ignore. Moreover, in the context of this study, all six models cannot be used as models of choice and together with the rejection of Litecion at 1% level, further tests are warranted. Since the FZL is elicitable and has a consistent scoring function; the models can be ranked in order of predictive ability. We show the MDM test of EPA in Table 2.

Table 2. Multivariate Diebold-Mariano (2012) test of model equal predictive accuracy

Cryptocurrency	α	W	p-value
Bitcoin	1%	44.213	1.00
	2.5%	75.524	1.00
Ethereum	1%	37.979	1.00
	2.5%	58.048	1.00
Litecoin	1%	**-429.38**	**0.000**
	2.5%	777.07	1.00
Ripple	1%	37.965	1.00
	2.5%	111.92	1.00
Das	1%	25.211	0.9999
	2.5%	44.686	1.00
Monero	1%	23.129	0.9997
	2.5%	42.397	1.00
Steller	1%	45.452	1.00
	2.5%	341.43	1.00

W is the MDM test statistic. MDM does not reject the null hypothesis of EPA for all cryptocurrencies at all conventional levels of significance except for Litecoin (LTC) 1% level (in boldface).

4.2. MAE ranking of univariate GAS FZL forecasts per distributional innovation

Following Blazsek and Hernández (2018), we calculate the MAE for each model across the samples at their respective α levels to rank the models, which is presented in Table 3. Models with the least MAE values are preferred. From Table 3, ALD emerged as the best model with the least MAE at both $\alpha = 0.01$ and $\alpha = 0.025$ levels. This confirms the study of Taylor (2019) which contends that the ALD is appropriate to jointly estimate and forecast dynamic models of VaR and ES. The robustness of ALD is also confirmed in this study as did Kotz et al. (2012) and Zhao et. al. (2015) that ALD is the most suitable skewed simplification of classical Laplace law. Nevertheless, the SNORM also appeared as the best model in a few instances at both $\alpha = 0.01$ and $\alpha = 0.025$ levels. AST and AST1 have the same MAE which is the highest across the board. The results for AST and AST1 provides evidence in support of Zhu and Galbraith (2010) study which indicates that the equality of the relative frequency of extreme returns in left tails (losses) and right tails (gains) of asset returns can be established using AST class of distributions. This implies that the returns of cryptocurrencies cannot be distinguished by virtue of left and right tail behaviours where one is thin and the other heavy in VaR and ES estimates. Our study shows the ALD and in some cases SNORM models as candidates for backtesting the FZL forecast as shown in Table 3.

Table 3. MAE ranking of univariate GAS FZL forecasts per distributional innovation

	α = 1%		α =2.5%	
	MAE	**Rank**	**MAE**	**Rank**
Bitcoin (BTC)				
snorm	3.1607	2	2.9999	2
std	3.9029	3	3.3759	4
sstd	3.9763	4	3.3655	3
ast	5.8154	5	4.2653	5
ast1	5.8154	5	4.2653	5
ald	**2.6573**	**1**	**2.8173**	**1**
Ethereum (ETH)				
snorm	**1.7797**	**1**	**1.9538**	**1**
std	3.0435	3	2.7099	3
sstd	3.0859	4	2.6973	2
ast	5.2219	5	3.8057	5
ast1	5.2219	5	3.8057	5
ald	2.1056	2	2.8195	4
Litecoin (LTC)				
snorm	2.1092	2	2.2452	2
std	2.8942	4	2.5461	4
sstd	2.8028	3	2.4974	3
ast	5.8835	5	4.3849	5
ast1	5.8834	5	4.3848	5
ald	**1.9575**	**1**	**2.1843**	**1**
Ripple (XRP)				
snorm	4.4938	5	3.5477	5
std	2.4745	3	2.5393	3
sstd	2.4007	2	2.4929	2
ast	3.4519	4	3.2062	4
ast1	3.4519	4	3.2062	4
ald	**2.1808**	**1**	**2.4527**	**1**
Das (DASH)				
snorm	2.900	3	2.7387	3

std	2.8931	2	2.759	4
sstd	2.9778	4	2.7259	2
ast	4.113	5	3.2817	5
ast1	4.113	5	3.2817	5
ald	**2.1163**	**1**	**2.2972**	**1**
Monero (XMR)				
snorm	2.200	2	2.203	2
std	2.3881	4	2.4007	4
sstd	2.3862	3	2.3839	3
ast	3.7679	5	3.1475	5
ast1	3.7678	5	3.1475	5
ald	**1.7868**	**1**	**2.1214**	**1**
Steller (XLM)				
snorm	1.4993	2	**1.6549**	**1**
std	2.098	4	2.2129	4
sstd	2.0391	3	2.1228	3
ast	3.5717	5	3.084	5
ast1	3.5717	5	3.084	5
ald	**1.4405**	**1**	1.7435	2

Best models are in boldface.

4.3 Backtesting and model ranking of FZL function

We implement the UC, CC, DQ, and QL backtest on the FZL forecast of the best performing models (ALD and SNORM) selected by MAE for each cryptocurrency at $\alpha = 0.01$ and $\alpha = 0.025$ thresholds. The null hypothesis of a correctly specified forecast model for the FZL at respective α levels[13] is evidenced by all the backtesting models. The UC, CC, and DQ tests statistics provide p-values whiles QL gives loss values. Models with p-values of UC, CC, and DQ test statistics close to unity are preferred whereas models with lower Quantile loss are preferred. Table 4 provides the backtesting results.

From Table 4, all models at both $\alpha = 0.01$ and $\alpha = 0.025$ of cryptocurrencies are accepted by UC and CC test as adequate and correctly specified and hence can adequately estimate and forecast FZL. However, the results of the DQ test reject all the models as adequate and correctly specified even if it is highly accepted by UC and CC test. This confirms the robustness of DQ test as claimed by Braione and Scholtes (2016) that DQ takes into account a more general temporal dependence between the series of violations.

To estimate the single value FZL for cryptocurrency tail risk and capital adequacy comparison, models that are correctly specified by at least one of UC, CC, and DQ are chosen for further analysis. However, models at risk levels $\alpha = 0.01$; $\alpha = 0.025$ that are not accepted by any of UC, CC, and DQ test as correctly specified is omitted from further analysis. In this study, since both UC and CC accepts the models at their respective α levels as correctly specified, both the 1% and

2.5% FZL forecast are used to estimate the single value FZL for the risk comparison in the cryptocurrency market.

Following the Basel III requirement, we compare the 1% model to 2.5% model of each cryptocurrency by computing QL ratios ($QLR = QL_{1\%}/QL_{2.5\%}$) of FZL forecast obtained from backtesting results in the last column. As Ardia et al. (2016c; 2018) notes, the 1% model outperforms the 2.5% if $QLR < 1$ and vice versa. The QL ratios in Table 4 depicts that except for Bitcoin (2%), and Das (1%) in favour of the 2.5% FZL model, the 1% FZL models outperform the 2.5% FZL models ranging from 9% to 61% in five cryptocurrency markets.

Table 4. Backtesting results of selected univariate GAS FZL models

Cryptocurrency	Distribution	α	UC	CC	DQ	QL	QL ratio	QL ratio (%)
Bitcoin	ald	1%	2.45(0.12)	2.70(0.26)	284.75(0.00)	0.587981		-0.02*
	ald	2.5%	2.51(0.11)	3.53(0.17)	251.72(0.00)	0.575433	1.021806	
Ethereum	snorm	1%	0.35(0.55)	0.38(0.83)	200.09(0.00)	0.096554		0.38
	snorm	2.5%	0.28(0.59)	0.55(0.77)	187.04(0.00)	0.155926	0.619229	
Litecoin	ald	1%	1.19(0.27)	1.36(0.50)	471.52(0.00)	0.293236		0.09
	ald	2.5%	0.02(0.87)	0.36(0.83)	226.57(0.00)	0.323699	0.905891	
Ripple	ald	1%	1.19(0.27)	5.13(0.08)	278.39(0.00)	0.075128		0.46
	ald	2.5%	1.89(0.17)	1.98(0.37)	118.30(0.00)	0.138782	0.541342	
Das	ald	1%	0.34(0.56)	0.45(0.79)	307.06(0.00)	0.410231		-0.01*
	ald	2.5%	0.86(0.35)	1.57(0.46)	220.21(0.00)	0.404281	1.014719	
Monero	ald	1%	2.45(0.12)	2.70(0.25)	500.14(0.00)	0.20253		0.39
	ald	2.5%	0.34(0.56)	0.91(0.63)	304.48(0.00)	0.333679	0.606961	
Steller	ald	1%	0.001(0.98)	6.79(0.03)	83.57(0.00)	0.041684		0.61
	snorm	2.5%	5.63(0.02)	5.66(0.06)	69.24(0.00)	0.106585	0.39109	

**Negative percentage indicates the 2.5% FZL model outperforms the 1% FZL model.*

4.4 Characteristic FZL estimates for 1% and 2.5% univariate GAS models

To compare the capital requirement, tail risk, and riskiness of the seven cryptocurrencies, we use the characteristic FZL (CFZL) values (single values of FZL) estimated from the unconditional[14] location parameters from the OOS FZL forecast of the best performing models. The characteristic FZL values in Table 5 are ranked in ascending order per their magnitude with the least inverse CFZL ranked 1 and 7 for the most negative. Smaller CFZL values are preferred to larger values. Table 5 shows that the capital required to absorb losses at the 1% risk threshold is least for Ethereum, followed by Steller, Monero, Das, Litecoin, Bitcoin and largest for Ripple. This implies that Ethereum has the least risk profile among the cryptocurrencies whiles Ripple is the riskiest currency at $\alpha = 0.01$ level.

However, at the 2.5% risk threshold Steller requires the least capital to absorb losses followed by Ethereum, Monero, Das, Litecoin, Bitcoin, and largest for Ripple. Which implies that at $\alpha = 0.025$ risk level, Steller is the least risky currency among the cryptocurrencies whiles Ripple is the riskiest. The results show that, Monero, Das, Litecoin, Bitcoin, and Ripple maintains their third, fourth, fifth, sixth, and seventh positions respectively at both 1% and 2.5% risk thresholds whiles Ethereum and Steller alternate the first and second positions. Our results suggest that at 1% and 2.5% risk levels, Ethereum and Steller are the safest cryptocurrencies which provides evidence against the study of Gkillas and Katsiampa (2018) which reported Litecoin and Bitcoin as cryptocurrencies with the least risk profiles adopting the extreme value technique to estimate VaR and ES of five cryptocurrencies.

Following the results of our study, we argue that potential cryptocurrency investors, hedge funds, market makers and traders can follow the rankings in this study to enhance their trading and investment strategies to maximize utility whiles minimizing risk since the extreme volatility of cryptocurrencies turns them a really risky assets. Our study confirms the study of Patton et al. (2019) which argues that FZL values drawn from both VaR and ES provides a better empirical risk modeling than either VaR and ES separately. Our approach provides coherent risk measure estimates which can be used by market participants interested in cryptocurrency markets for an effective portfolio optimization and risk management.

Table 5. Characteristic FZL forecast values for selected distributional innovations

α = 1%					α =2.5%			
Cryptocurrency	**Innovation**	**CFZL**	**Rank**		**Cryptocurrency**	**Innovation**	**CFZL**	**Rank**
Ethereum	snorm	-1.2098	1		Steller	snorm	-1.1231	1
Steller	ald	-1.5654	2		Ethereum	snorm	-1.3340	2
Monero	ald	-1.9100	3		Monero	ald	-2.2004	3
Das	ald	-1.9913	4		Das	ald	-2.2727	4
Litecoin	ald	-2.0031	5		Litecoin	ald	-2.3274	5
Bitcoin	ald	-2.5041	6		Bitcoin	ald	-2.7877	6
Ripple	ald	-2.5631	7		Ripple	ald	-2.8269	7

CFZL – characteristic FZL forecast

5. Conclusion and policy implication

The study applies the FZL function drawn from both VaR and ES to measure tail risk among seven cryptocurrencies (Bitcoin, Ethereum, Litecoin, Ripple, Das, Monero, and Steller) for the period August 2015 to February 2019. We capture heavy tails and volatility clusters that characterize cryptocurrency returns applying six different asymmetric distributions such as SNORM, STD, SSTD, AST, AST1, and ALD to the univariate GAS framework. We first of all, applied MDM test to the six distributions to check the equal predictive accuracy (EPA) of the models. Results from MDM test of equal predictive accuracy (EPA) failed to reject the null hypothesis of EPA at all conventional levels of significance for all cryptocurrencies except Litecoin at 1% level. Since all six models cannot be used as models of choice and together with the rejection of Litecoin at 1% level, we calculate the MAE for each model across the samples to rank the models. Results from MAE shows the ALD and in some cases SNORM as the best models with the least MAE at both $\alpha = 0.01$ and $\alpha = 0.025$ levels. These two models were chosen as candidates for backtesting the FZL forecast.

As a typical out-of-sample forecasting procedure, we apply the UC, CC, DQ, and QL test to the best performing models (ALD and SNORM) selected by MAE to backtest and check the adequacy of the models. The UC and CC test accepted the ALD and SNORM models at their respective α levels as adequate and correctly specified except DQ test. As required by Basel III, we compared the 1% models to 2.5% models by computing QL ratios. The QL ratios depict that the 1% models outperforms the 2.5% models. Finally, we estimated the characteristic FZL value to compare the capital adequacy and riskiness of the cryptocurrencies. Results from the characteristic FZL show that, at both 1% and 2.5% risk thresholds, Ethereun and Steller has the least risk profile followed by Monero, Das, Litecoin, Bitcoin, and Ripple with the largest risk profile. In a nutshell, our study suggests that at $\alpha = 0.01$ and $\alpha = 0.025$ risk levels as required by BIC (2013), Ethereum and Steller requires the least capital to absorb losses followed by Monero, Das, Litecoin, Bitcoin, and Ripple.

The extreme price volatility of cryptocurrencies coupled with their exposure to tail risk makes them really risky currencies since the financial impact of tail risk could be large despite their small likelihood of occurrence. Hence, the application of FZL function for joint VaR and ES to cryptocurrencies provides coherent risk measure estimation for an enhanced investment and trading strategies for cryptocurrency portfolio optimization. Our study goes beyond prior studies which mainly described the volatility dynamics of cryptocurrencies (Gkillas, Bekiros, and Siriopoulos, 2018; Trabelsi, 2018; Borri, 2019; Huynh et al., 2018; Huynh, 2019) by estimating risk measures which is the main contribution of this study. We provide evidence in support of the study of Patton et al. (2019) which argues that FZL values drawn from both VaR and ES provide a better empirical risk modeling than either VaR and ES separately. This study focused on the tail risk of seven cryptocurrencies using FZL Function for joint VaR and ES. Future studies can replicate this study by extending the enquiry into several cryptocurrencies and other financial instruments to broaden our understanding of the volatility of the financial instruments.

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Data availability statement

- Data for this manuscript is available and will be provided upon request.

Footnotes

[1] Cryptocurrencies are based on cryptographic proof which provides many advantages over traditional payment methods including lower transaction costs, high liquidity, and anonymity (Fantazzini et al., 2016).

[2] The initial Basel III was expected to be implemented by 1st January, 2019 while the *final* Basel III minimum requirements are expected to be implemented by 1 January 2022 and fully phased in by 1 January 2027 (Patton, Ziegel, & Chen, 2019), https://www.bis.org/press/p181004.htm.

[3] As noted by Patton, Ziegel, and Chen (2019) a risk measure is elicitable if it has a loss function such that expected loss is minimized by the risk measure. Fissler, Ziegel, and Gneiting (2015) argue that elicitability is important because it allows for model estimation, selection, forecast ranking and comparison.

[4] The Basel Accords (currently the Basel III Accords) regulatory process requires financial institutions to meet required capital by correctly assessing and predicting their VaR and ES using state-of-the-art risk systems (Ardia, Boudt, and Catania, 2016; 2018).

[5] EVT provides a firm theoretical foundation for building a statistical model describing extreme events (Ramadhani, Nurrohmah and Novita, 2017).

[6] Note that, as mentioned by Creal et al. (2013), the scaling matrix $S_t\left(\theta_t\right)$ can take the form of an identity matrix, inverse fisher information matrix, or pseudo inverse square root, leading to different types of GAS models. Following Troster et al. (2018), the study looks at only the inverse fisher information matrix.

[7] There is almost no end to the list of non-Gaussian distributions one can choose from. However, for the purpose of this study, we employ SNORM, STD, SSTD, AST, AST1, and ALD. These distributions, together with GAS, capture skewed, fat-tails and volatility clustering which is a characteristic of cryptocurrencies returns (Troster et al., 2018).

[8] To evaluate FZL model specification, the risk level α is typically set to $\alpha = 0.01$; $\alpha = 0.025$ for which asset's loss is expected to be exceeded for a given period of time (see BIC, 2013; Ardia, Boudt, and Catania, 2016; 2018).

[9] The UC test verifies the correct coverage at the left-tail of the marginal distribution of returns.

[10] The CC test analyses the conditional density of returns.

[11] The DQ test simultaneously tests for conditional and unconditional coverage and is more robust than UC and CC.

[12] The QL function is crucial for selecting the best FZL model if two models achieve CC/UC.

[13] For consistency all tests are carried out at the same α levels in agreement with 99 and 97.5 percentiles of the FZL forecasts.

[14] Conditional parameters are the case in which the scale parameter is set to be time-varying in the typical GAS models used for FZL estimation and forecasting. The unconditional parameters have not time-varying assumptions.

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