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# Addition and Subtraction of Numbers Through Visualisation Using the Number Grid Technique Among Primary School Students

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## Abstract

> This research investigates the effectiveness of a spatial imagery-based method, utilizing a number grid, as an attempt to teach addition and subtraction to elementary school students. Rooted in Bruner's cognitive development theory, the study considers enactive, iconic, and symbolic stages, introducing the number grid through a series of interventions. Additionally, the study seeks to identify the common errors that students make while engaging in the learning process through visualisation using number grid. The qualitative approach, involving observations, interviews, and assessments, assesses the performance of eleven home-schooled students of grades two and three. Results reveal a progression in proficiency from enactive to symbolic stages, with a distinct challenge observed in subtraction at the iconic stage. Notably, grade three students, acquainted with the standard algorithm, exhibited potential resistance to alternative strategies.

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## 1. Introduction

Numeracy has many meanings for different purposes across the domains, in terms of number sense, Numeracy is defined as the ability to understand and use numbers (Dieckmann, 2008).

Adding and subtracting numbers are considered numeracy skills (Heirdsfield & Cooper, 2002). Teaching addition and subtraction is important for students to properly understand and practice math facts, which helps them memorize basic math facts (Greene, 1985). Teaching addition and subtraction is generally done using the Standard algorithm for adding and subtracting numbers. The standard algorithm for adding and subtracting numbers is by writing numbers in columns and using carrying and borrowing. Other than the standard algorithm, there are other techniques like the use of diagrams as a form of mediational representation (Gutstein & Romberg, 1995) to introduce addition and subtraction of numbers. Abacus is one of the methods for teaching addition and subtraction (Alhassan et al., 2018). Abacus is a mechanical appliance that makes counting easy, it was known already in Babylonia around 3000 years ago (Wierzbicki, 2014). Anam et al., (2020) claimed that the application of the Abacus learning model, supported by teaching aids, demonstrated a significant enhancement in students' Mathematics performance, particularly in addition, subtraction, and mixed calculations at Elementary School Kebraon II in Surabaya, Indonesia. Notably, there was a 60% increase in student performance, as indicated by the percentage of students who achieved mastery learning in the first cycle and an 86% increase in the second cycle.

Children who employ alternative approaches instead of algorithms can effectively apply these methods in various scenarios, as evidenced by the significantly higher success rate of students in the alternate strategy groups when solving extension problems compared to students in the algorithm group (Carpenter et al., 1998). Febrianti & Purwaningrum (2021) claimed that students will build upon their existing ideas to facilitate the creation of new ones. This connection between prior knowledge and novel concepts enhances understanding and fosters the generation of fresh ideas during the thought process. Children (as well as adults) can solve problems and give answers that are counter intuitive and meaningless while using the standard algorithm (Gutstein & Romberg, 1995), therefore the alternative approach to teach addition and subtraction of numbers was planned.

Charles (2020) asserted that the initial guidance for teaching numeracy should prioritize the use of tangible models, illustrations, and approaches rooted in concepts like place value, the properties of mathematical operations, and the connections between addition and subtraction and alternative computation strategies promote flexibility. Visual aids like images and charts can serve as a helpful support in prompting conceptual understanding and aiding the process of integration (Rittle-Johnson & Koedinger, 2005). In this research study, the instruction of addition and subtraction employs a visualisation approach that utilizes a number grid. Mazur (2004) defined visualisation as something that we have the mental equipment to visualize, we picture onto the mind's screen. Presmeg (2006) also defined Visualisation as a process of constructing and transforming both visual, mental imagery and all of the inscriptions of a spatial nature that may be implicated in doing mathematics. Also, Van Hiele's pioneering work in the realm of visualizing mathematics, as documented by Yelland and Masters (1997), emphasizes that successful students are not those who memorize isolated facts but those who possess the capacity to establish intricate networks of relationships that interconnect geometric concepts and processes.

A study by Salam et al. (2019) revealed an interaction effect between instructional models and spatial intelligence. Students with high spatial intelligence demonstrated better results with integrative instruction compared to direct instruction. Spatial intelligence as defined by Gardner (2011) is the ability to interpret visual information and recreate experiences without direct physical stimuli.

This research follows Bruner's cognitive development theory, which posits that children learn concepts in a subject by progressing through the stages of enactive, iconic, and symbolic representation (Conway, 2007). Bruner's cognitive development theory can enhance the effectiveness of learning the focal subject matter while promoting active student involvement in classroom activities, thereby facilitating their journey towards conceptual exploration (Budiman et al., 2023). While several studies have applied Bruner's theory to instructing addition and subtraction with the use of object imagery, spatial imagery has yet to be extensively explored in this context. Object imagery refers to how we represent the way things look, like their shape, color, and texture. On the other hand, spatial imagery is about how we show the relationships between objects, where they are located in space, how they move, and other complex spatial changes (Kozhevnikov & Blazhenkova, 2012). Chrysostomou et al. claim that the higher the school teacher's tendency towards spatial imagery cognitive style, the more conceptual and flexible strategies they employ in algebraic reasoning and number sense tasks (2012). Many studies employing Bruner's theory have primarily utilized object imagery. In contrast, this study deviates by implementing spatial imagery through the use of a number grid drawn on the floor for teaching addition and subtraction.

Magnaye and Magnaye (2019) emphasize the role of number grids in facilitating precise numerical alignment, thereby fostering a comprehensive understanding of concepts like place value and regrouping. Additionally, these grids serve as visual aids, contributing to the development of cognitive learning skills among students and enhancing their capacity for active learning.

A parallel viewpoint was presented by Lacey (1998), who terms these grids as "number squares." Lacey delves into a detailed exploration of how these grids can effectively heighten students' grasp of number sense. These scholarly perspectives highlight the importance of integrating number grids into educational approaches, offering a means to enrich mathematical understanding and promote cognitive development.

In this study, a series of interventions spanning five sessions were planned and executed. The initial session involved the number grid from 1 to 100 drawn on the floor. In the second session, paper-based number grids (1 to 100, 101 to 200, and up to 901 to 1000) were presented horizontally. Students, equipped with two beads, engaged in addition and subtraction operations within a selected grid. Simultaneously, they practiced spatial reasoning by placing a number between 1 and 100 in an empty grid. The third session featured a floor grid with only 1, 50, and 100 visibly marked. Students identified the positions of other numbers, physically walking the grid and applying rules for addition and subtraction. In the fourth session, students mentally visualized addition and subtraction operations on sequential grids (1 to 1000). Imagination replaced physical beads, emphasizing mental engagement. In the fifth and final session students formed a circle, sequentially contributing numbers (1 to 1000) for addition or subtraction.

Limited research has explored the application of spatial imagery techniques for teaching addition and subtraction through visualisation methods. Despite evidence suggesting that spatial imagery can have a favorable influence on number sense, as reported by Chrysostomou et al. (2012), there are notably fewer investigations that assess the differential impact of visualisation techniques across different grade levels and gender groups. Number grid is used in this study as a spatial imagery technique to understand how students navigate Bruner's cognitive stages of Enactive, Iconic and Symbolic while learning addition and subtraction of numbers through visualisation and to know the common errors done by students while adding and subtracting numbers using number grid.

### 1.1. Research Questions

1. How visualisation using number grid helps the students to learn addition and subtraction in this study?
2. What are the common errors done by students during visualisation using number grid?

## 2. Methods

The methodology employed in this study is a qualitative approach: The goal of the qualitative study is to provide a thorough explanation and profound comprehension of the phenomenon of interest, which is people's lived experiences in natural environments (Magilvy, 2003). Qualitative data was collected through observations, open-ended interviews, students' explanations, and the assessment taken by the students.

### 2.1. Participants selection

In this exploratory study, we adopted a convenience sampling approach to gather insights. Convenience sampling was chosen due to its practicality and accessibility in obtaining data from a specific population of interest. The goal is to explore how visualisation techniques impact the learning of addition and subtraction numbers and common errors done by the students. The use of convenience sampling allowed for a quick and efficient data collection process, facilitating a preliminary understanding of the common mistakes and difficulties students face while using visualisation techniques to learn addition and subtraction of numbers. In this study, eleven students were selected from a homeschool (a parent-coordinated school) where parents lead the learning community. The group comprised six grade two students and five grade three students, including six female students and five male students. Piaget's stages of development include sensory-motor, pre-operational, concrete operational, and formal operational (Piaget, 1929). The selected age group students are in the concrete operational stage and can perform mathematical problems such as adding, subtracting, placing objects in order, and many other operations with concrete objects (Horn & Kincheloe, 2006).

### 2.2. Procedure

The study was conducted in five sessions, with each session lasting one and a half hours, taking place once a week. During each session, changes in students' learning and adaptation to the grid-based approach for addition and subtraction of whole numbers over time were observed and documented.

#### Enactive Stage

During the first session, a number grid spanning numbers 1 to 100 was drawn on the floor as shown in Fig. 1. Students were asked with visually inspecting the grid to ensure the sequential order of all the numbers. Subsequently, a specific rule was instructed to the students, describing the process of adding or subtracting a given number from the grid to another number within the grid, thereby generating a resultant number. In the third session, a number grid was drawn on the floor, however, not all the numbers were explicitly marked. Only numbers 1, 50, and 100 were written on the floor. Students were challenged to identify the positions of the remaining numbers within the grid and were encouraged to imagine themselves standing on a particular number within the grid. They were then prompted to apply the prescribed rules for adding and subtracting a specific number from the grid to reach the resultant number. To facilitate this process, students were asked to physically walk on the grid while adhering to the established rules to achieve the desired results.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 11 | 21 | 31 | 41 | 51 | 61 | 71 | 81 | 91 |
| 2 | 12 | 22 | 32 | 42 | 52 | 62 | 72 | 82 | 92 |
| 3 | 13 | 23 | 33 | 43 | 53 | 63 | 73 | 83 | 93 |
| 4 | 14 | 24 | 34 | 44 | 54 | 64 | 74 | 84 | 94 |
| 5 | 15 | 25 | 35 | 45 | 55 | 65 | 75 | 85 | 95 |
| 6 | 16 | 26 | 36 | 46 | 56 | 66 | 76 | 86 | 96 |
| 7 | 17 | 27 | 37 | 47 | 57 | 67 | 77 | 87 | 97 |
| 8 | 18 | 28 | 38 | 48 | 58 | 68 | 78 | 88 | 98 |
| 9 | 19 | 29 | 39 | 49 | 59 | 69 | 79 | 89 | 99 |
| 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |

**Fig. 1.** Number grid from 1 to 100

Instructions given to the students:

- To add ‘1’ move down to the next tile
- To subtract ‘1’ move up to the next tile
- To add ‘10’ move right to the next tile
- To subtract ‘10’ move left to the next tile

Students were involved in active participation by following the instructions given to them. For example, to add 11 to 79, the student moved right to the next tile that is 89, and down once to the next tile that is 90.

Iconic Stage

In the second session, we introduced a set of paper-based number grids ranging from 1 to 100, 101 to 200, and so forth, up to 901 to 1000. These grids were arranged in a horizontal order. Each student was provided with two beads and instructed to place one bead on any number within these ten grids. They were then asked to perform specific addition and subtraction operations using the selected number and the second bead, aiming to reach a resultant number within the same grid. Moreover, in the same session, students were presented with an empty grid on the floor where only the numbers 1 and 100 were marked. They were tasked with locating the approximate placement of a number between 1 and 100 within this vacant grid, aiming to enhance their spatial reasoning abilities. The fourth session commenced with a revisit to the iconic stage, where ten sequential grids spanning numbers 1 to 1000 were arranged systematically. However, unlike previous sessions, students were not provided with physical beads to place on the grid for addition and subtraction operations, as they had done in the initial phase of the intervention. Instead, students were encouraged to employ their imagination. They were asked to visualize the act of placing a bead on a specific number within the grid and then proceed to mentally perform addition and subtraction operations, utilizing the predefined rules, with the objective of reaching a resultant number. The iconic stage acts as a bridge between the enactive and symbolic stages. For example, to subtract 202 from 422, a student moved the bead 20 times right from 422 reaching 222 and moved two times up getting the answer as 220. As students progress through the Iconic stage, they gradually move from concrete representations to more abstract visual thinking.

Symbolic Stage

During the fifth and concluding session, students were asked to perform the operations orally by visualizing the grid and asked to explain the procedure they followed. One example is to subtract 23 from 156, the explanation given was "*From 156, I moved left twice reaching the number 136, and moved up thrice getting the answer 133*" and an activity was given in the same session where students formed a circle where one student would contribute a number ranging from 1 to 1000. The subsequent student in the circle was tasked with subtracting 30 from the given number and passing a new number to the following student. This activity was systematically repeated using various numbers for addition and subtraction operations and was executed as part of the Symbolic stage in the intervention.

Finally, a test was conducted to assess their learning. The assessment was structured into two parts - Addition and Subtraction, each containing four questions:

- The first question tests the students' number sense, likely evaluating their basic understanding of numbers and their relationships based on the number grid that is, it assesses the students' understanding of seriation.
- The second question is focused on assessing the iconic level of understanding.
- The third question contains two sub-questions. These questions assess the symbolic level, which likely requires a deeper understanding of mathematical symbols and their application. Additionally, these questions involve plotting on the number grid to test the iconic level within the symbolic understanding.
- The fourth question examines the visualisation. In this question, students are required to mentally visualize the grid and execute the provided operations based on the given starting number and directions on the grid, ultimately arriving at the result without physically using the grid.

**Table 1** Distribution of Questions Across Dimensions

| S. No. | Q. No. | Dimensions |
|---|---|---|
| **1** | 1 | Number sense (ability) |
| **2** | 2 | Iconic level of understanding (cognitive stage) |
| **3** | 3a(i) | Symbolic level of understanding (cognitive stage) |
| **4** | 3a(ii) | Iconic level of understanding (cognitive stage) |
| **5** | 3b(i) | Symbolic level of understanding (cognitive stage) |
| **6** | 3b(ii) | Iconic level of understanding (cognitive stage) |
| **7** | 4 | Visualisation (ability) |

These questions were designed to evaluate the student's understanding in number sense, iconic and symbolic level of understanding, and finally visualisation in the context of both addition and subtraction using the number grid. This assessment not only evaluates their arithmetic skills but also their ability to navigate through cognitive levels of understanding mathematical operations through the number grid.

## 2.3. Data collection

Qualitative data was collected through participant observation in each session. This involved noting the progress made by students, the challenges encountered, and the explanations provided by students for their responses during grid-based activities. These observations allowed for tracking the progress of individuals and overall progress over time.

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Data collected from the assessment provided information about students' performance levels. The design of the assessment was formulated to evaluate the students' comprehension in number sense, iconic and symbolic level of understanding, and visualisation. Its purpose is to systematically track and assess students' progress and understanding of learning, particularly through the visualisation technique employing a number grid.

## 2.4. Analyses

After conducting the assessment, the obtained results were encoded using a binary code system, enabling a structured representation of the data. The analysis involved calculating the percentage distribution within each dimension of number sense, iconic, symbolic, and visualisation in both addition and subtraction. This analysis provided a comprehensive understanding of students' learning progress at different stages of their learning process. This analysis not only highlighted the areas where students excelled but also pinpointed the areas that required further attention or clarification in their learning process. The data was further analyzed by considering the results based on the student's grades and gender. This segmentation enabled a more detailed understanding of the performance levels within each dimension of number sense, iconic, symbolic, and visualisation for both addition and subtraction, specific to each grade level and gender.

Initially, errors made by each student were cataloged across all dimensions, namely number sense, iconic, symbolic, and visualisation. In each dimension, if a mistake occurred two or more times, it was identified as a common mistake specific to that dimension to discern recurring mistakes or challenges faced by the students. These error patterns provided crucial insights into the challenges and recurring mistakes that students encountered while performing mathematical operations on the grid. The identification and categorization of these errors were essential in uncovering specific areas where students faced difficulties, whether it was related to misunderstanding certain mathematical concepts, struggling with the grid-based method, or encountering common misconceptions.

## 3. Results

### 3.1. Dimension-wise findings

The dimension-wise findings from our research shed light on the varying levels of proficiency among students. In the realm of number sense, 63.63% of students exhibited competence, reflecting a solid foundation in the enactive stage where students engage with direct experiences. Transitioning to iconic stage problems, the 60.6% success rate suggests a slightly lower but still significant grasp, signifying progress in using iconic representations. The subsequent advancement to symbolic stage problems revealed a heightened proficiency of 70.45%, indicating an enhanced capability in manipulating abstract mathematical representations, in line with Bruner's symbolic stage. However, in the context of visualisation problems, incorporating Bruner's emphasis on the importance of visualisation in learning, the success rate slightly decreased to 59.09%, with subtraction problems within this category notably affecting the overall score. Particularly, visualisation problems related to subtraction presented the most challenging area with a success rate of 45.45%, highlighting potential obstacles in transitioning to the symbolic representation of subtraction. The overall score of 63.63% underscores a generally positive performance among students.

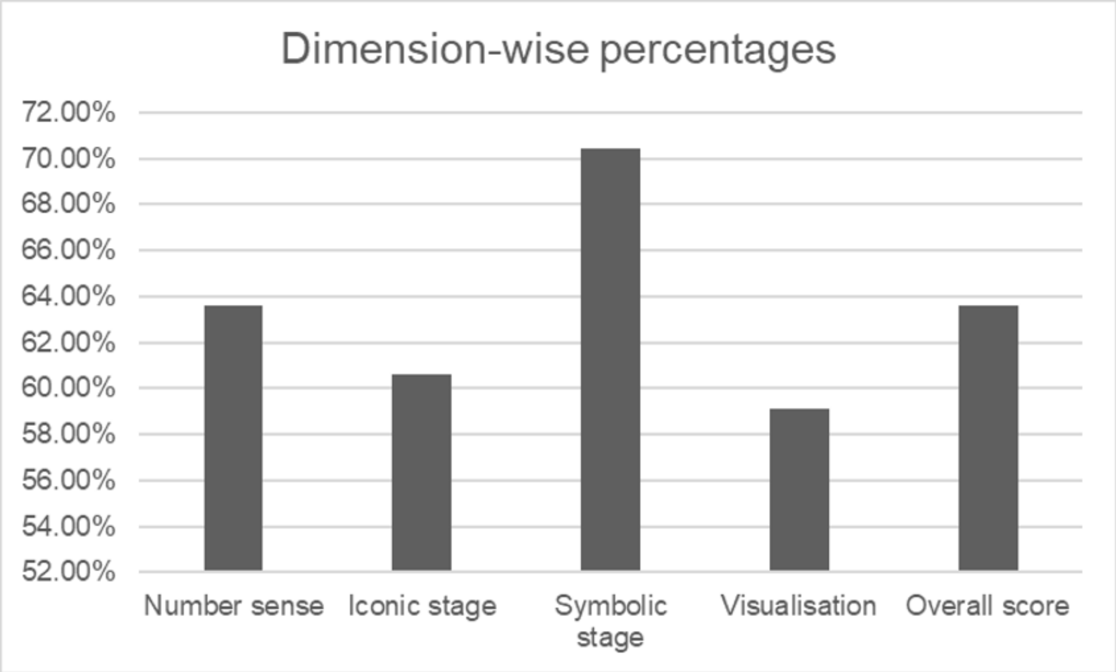


**Fig. 2.** Dimension-wise percentages

### 3.2. Addition problems and Subtraction problems

In the domain of number sense questions, 63.63% of students successfully answered both addition and subtraction problems. During the iconic stage, 69.69% demonstrated proficiency in answering addition problems, while only 51.51% achieved the same for subtraction problems. Progressing to the symbolic stage, 77.27% of students answered addition questions, whereas 63.63% answered subtraction questions. In the context of visualisation problems, 72.72% of students exhibited competence in answering addition questions, in contrast to a lower 45.45% success rate in responding to subtraction questions. This aligns with the outcomes reported in the study by Shinskey et al. (2009), where it was observed that in non-symbolic arithmetic, addition outcomes were more accurate than subtraction outcomes. An evident progression from the enactive stage to the iconic and symbolic stages of learning is discernible in students' handling of addition problems. However, in the case of subtraction, a decline in performance is notable, particularly at the iconic stage. This suggests a difficulty for students in advancing beyond the enactive stage when dealing with subtraction, resulting in challenges related to visualisation and execution of subtraction within the framework of the number grid. These results answer the first research question which highlights that Visualisation using number grid is more effective in addition of numbers than subtraction of numbers.

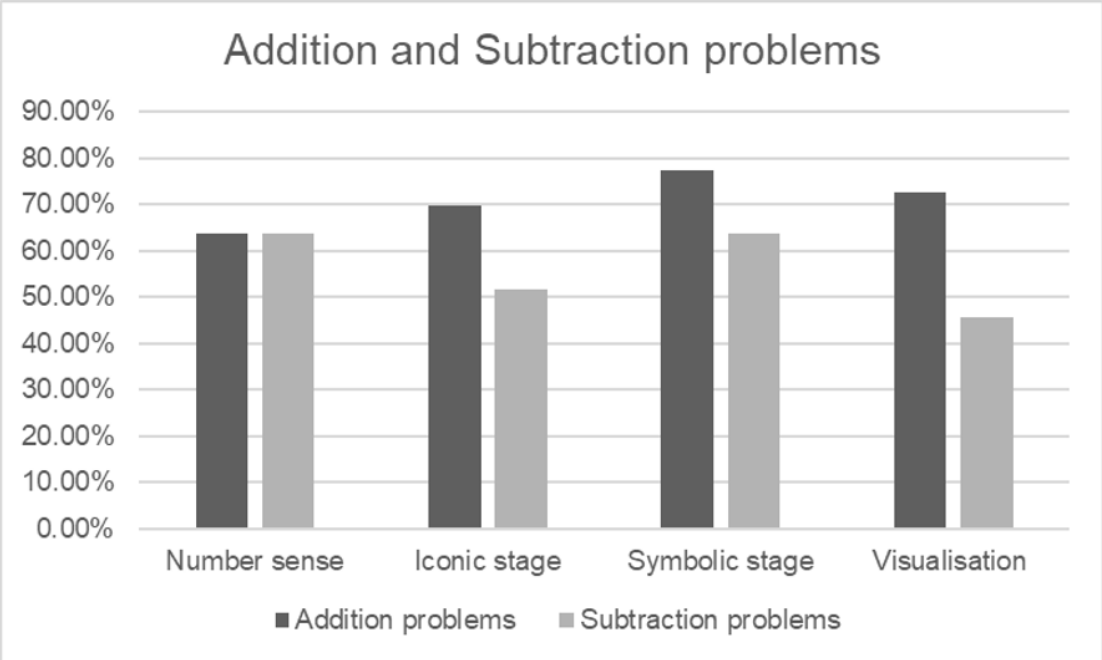


**Fig. 3.** Addition problems and Subtraction problems

### 3.3. Grade and gender-wise performance

In the context of number sense questions, 50% of grade two students answered correctly, while 80% of grade three students provided accurate responses. For iconic stage questions, 66.67% of grade two students and 53.33% of grade three students answered correctly. Moving on to symbolic stage questions, 66.67% of grade two students and 75% of grade three students responded accurately. Regarding visualisation questions, 66.67% of grade two students answered correctly, whereas only 50% of grade three students achieved accuracy. This pattern in symbolic stage and visualisation aligns with Charles's claim that introducing a standard algorithm makes it challenging for students to revert to alternative strategies (Charles, 2020), as evidenced by our study's grade three participants who were introduced to the standard algorithm before this study was conducted.

Overall, 64.28% of grade two students and 62.86% of grade three students answered all questions correctly.

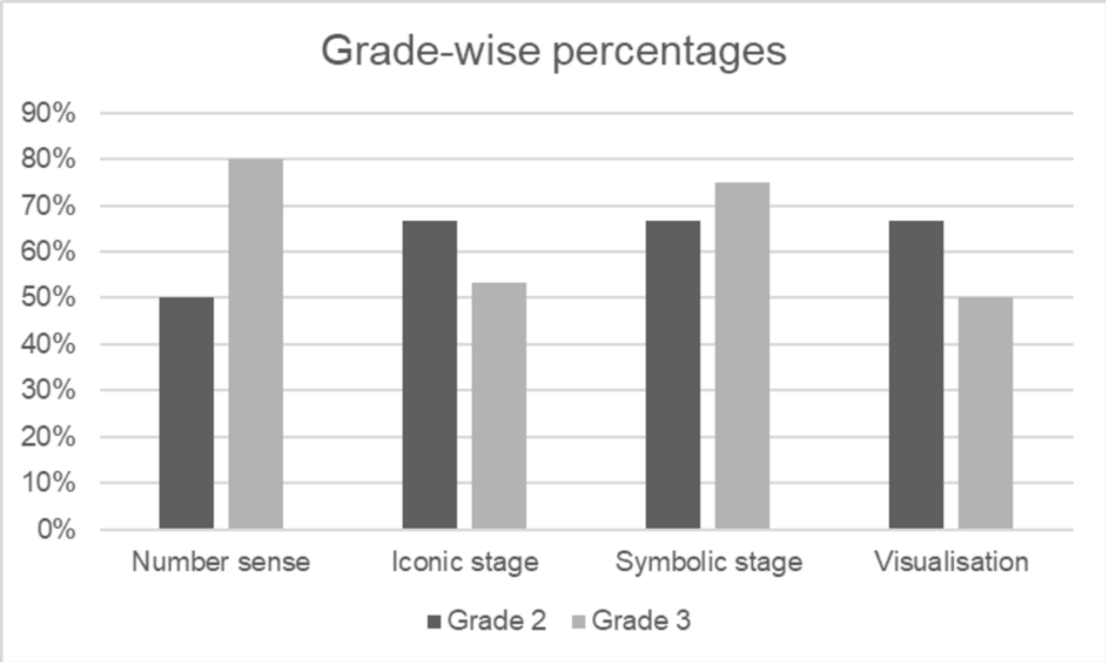


**Fig. 4.** Grade-wise percentages

In the context of number sense questions, 60% of male students and 66.67% of female students answered correctly. For iconic stage questions, 56.67% of male students and 63.89% of female students answered correctly. Moving to symbolic stage questions, 75% of male students and 66.67% of female students answered correctly. Regarding visualisation questions, 50% of male students answered correctly, while 66.67% of female students answered correctly. Overall, 61.42% of male students and 65.48% of female students answered all questions correctly.

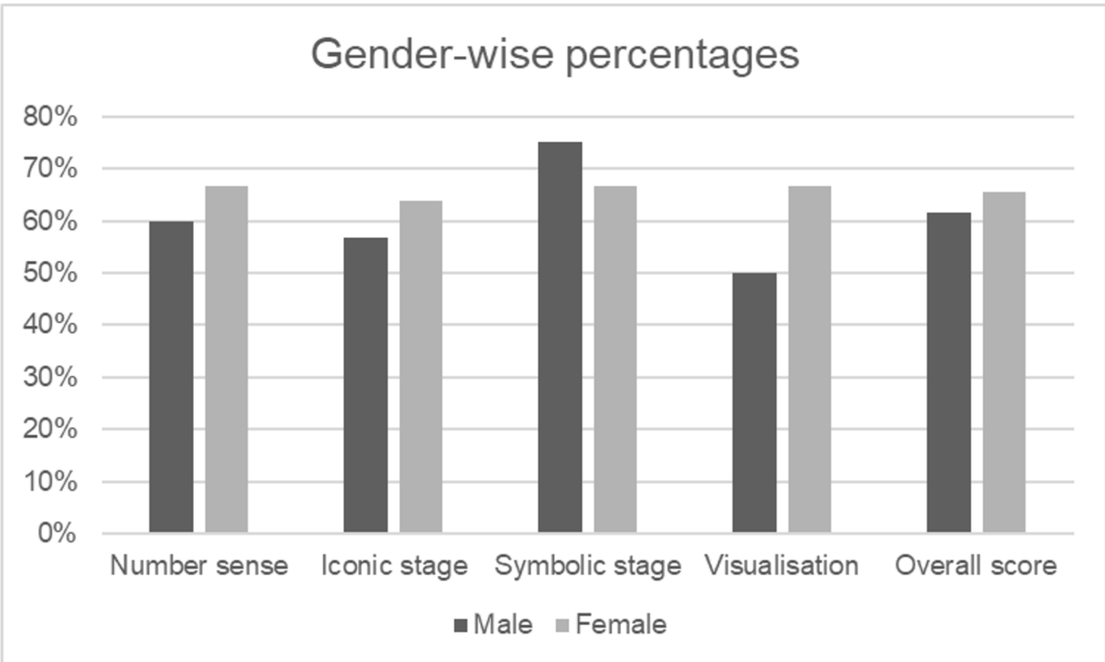


**Fig. 5.** Gender-wise percentages

### 3.4. Error pattern

In the initial phase of understanding numbers, a common error pattern observed among the students pertained to confusion with adjacent numbers within the number grid. This challenge appeared to influence their ability to discern and navigate through numerical sequences accurately, perhaps as they are still in the process of acquiring the skills of seriation. Furthermore, during the iconic, symbolic stages and visualisation, it was noted that when confronted with multi-step problems requiring a three-step movement across the grid, particularly in cases involving three-digit numbers with distinctions at the hundreds, tens, and one's places, students frequently encountered difficulties. Notably, while many students successfully executed one or two steps correctly, a consistent pattern of errors emerged in the completion of the third step. This nuanced observation sheds light on specific areas of numerical processing where students may benefit from targeted interventions and instructional strategies to enhance their mathematical proficiency. These results answer the second research question, that points out a process of three or more steps to add or subtract using number grid is challenging to most students.

While this study offers valuable insights into the proficiency levels of students, it is imperative to acknowledge the inherent limitation of a small sample size (n=11). The restricted sample size limits the generalizability of our findings to a broader population. However, despite this limitation, this research serves as a foundational exploration, providing a nuanced understanding of students' performance in mathematical dimensions, grades, and genders. Future studies with larger sample sizes are recommended to validate and extend the implications of our preliminary findings.

## 4. Discussion

Students notably exhibited superior performance in addition, as evident from their heightened performance in the symbolic addition stage. This success in symbolic addition was associated with their adeptness at enactive and iconic stages. Conversely, there was a discernible lower performance in the visualisation stage for the subtraction operation, indicating a challenge in progressing beyond the enactive stage in subtraction problems. The difficulty in subtraction may be because of the reason that children do not retrieve independent subtraction facts that they have internalized, instead, they deduce differences based on their understanding of sums (Kamii et al., 2001). The operation of subtraction is widely dependent on the ability to add.

The difficulty in executing three or more steps may arise from challenges in sequential processing. Students may struggle to follow a step-by-step approach, leading to errors in the intermediate steps and ultimately affecting the accuracy of the result. This error may occur as students tend to overlook specific grid cells or neglect to explore all potential paths, resulting in incomplete or inaccurate solutions.

The grade two students consistently outperformed their grade three counterparts in iconic and visualisation stages. However, this pattern did not extend to the symbolic stage, as grade three students had been introduced to the standard algorithm for addition and subtraction. It is noteworthy that once students learn the standard algorithm, they may find it challenging to revert to alternate strategies (Charles, 2020). As mathematical operations are cognitive skills, the practice of one standard algorithm or approach may intervene with the learning of the new approaches (Takane,2013)

The recognition of a common challenge, observed in 73% students of both grades, in solving subtraction problems within the iconic stage, particularly those with more than three steps, underscores the difficulty in applying the grid rule under such circumstances.

Importantly, these observations outline trends within the current sample, and caution is warranted in generalizing these findings due to the limited participant pool. To establish more robust conclusions, further studies with larger samples are imperative. Despite this limitation, this research lays the groundwork for a nuanced understanding of students' performance across various mathematical dimensions, grade levels and genders.

### 4.1. Implications of the study

Moving forward, the insights gained from this study provide a solid foundation for future research endeavours with a broader scope, offering potential applications in using visualisation methods to teach addition and subtraction of numbers. Advancing through the learning stages, starting with hands-on experiences in the enactive stage, enhances the efficiency of transitioning to the iconic and symbolic stages of representing mathematical operations. The use of visualisation in the symbolic stage helps students develop formal thinking abilities.

The identified disparity between addition and subtraction performance suggests a need for future research to incorporate more targeted practice sessions focused on subtraction. This can help bridge the gap and enhance students' proficiency in subtraction operations using number grids.

Regular practice with a focus on multistep problems is essential. By incorporating diverse examples and encouraging active problem-solving, students can reinforce their understanding and develop greater confidence in tackling complex addition and subtraction scenarios.

Plans are underway to conduct more extensive studies involving larger and more diverse samples to corroborate and extend the observed trends and allow for a more in-depth exploration of the intricacies surrounding students' mathematical performance.

### 4.2. Limitations and Prospects for Further Study

This study, employing a convenience sample drawn from home-schooling students, acknowledges certain limitations that merit consideration. The relatively small sample size poses challenges for generalizability. Despite its constraints, this study serves as a foundational exploration, providing a nuanced understanding of students' performance across different dimensions, grade levels, and genders. Future research endeavors can aim to address these limitations by employing more comprehensive sampling strategies and exploring additional variables that may influence students' mathematical abilities. Through this iterative process, a more comprehensive understanding of the nuances in mathematical learning and performance can be achieved.

## Statements and Declarations

### Ethics approval

ER-23-11-02 obtained from the Ethics Committee of Prayoga Institute of Education Research.

### Consent to participate

Informed consent was obtained from the parents of all participants.

### Consent for publication

It is affirmed that the authors named above are the sole authors of the present manuscript, that this manuscript has not been published elsewhere and that this manuscript will not be submitted elsewhere until the Journal's editorial process is completed.

### Conflict of interest

On behalf of all authors, the corresponding author states that there is no conflict of interest.

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