Review of: "Mathematics Is Physical"

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There are two very interesting themes addressed by Biao Wu. The first one concerns the connection between physics and mathematics, while the second one focuses on the relationship between the discrete and the continuous, specifically, the countable and the uncountable. The author argues that physics influences mathematics through machines, and thus through the theory of computability. In particular, the author mentions the quantum computer as an example of how physics enhances and extends computation and mathematics. Biao Wu develops the relationship between the countable and the uncountable in the context of formal axiomatic systems, involving Gödel's theorems in the discussion. Since the concept of machine dominates both themes, it seems appropriate to begin my commentary on the article from this point.

In the abstract and throughout the text, Biao Wu states that it is important to recognize that the development of mathematics is fundamentally influenced by a basic fact: mathematicians and computers are physical objects subject to the laws of physics. While this statement is obviously true, it overlooks a crucial point that requires careful consideration: machines and humans are profoundly different. More specifically, there is nothing wrong with asserting that humans, or living systems in general, can be considered, from a biological point of view, as machines. It is however crucial that the nature of this machine is well-defined, and I shall briefly do that following Maturana and Varela work and that of Damasio.

A machine is a system composed of a certain number of components, each of which is in a specific relationship with other components. The set of relationships constitutes the organization of the machine. A given organization can be concretely realized with different components, as long as the organization is maintained. In other words, machines with the same organization can have different structures. For example, a tank level control system, while maintaining its organization and thus the relationships among its components, can be realized in many ways. It is important to note that the purpose of a machine does not belong to its organization, but rather to the domain in which the machine operates and to the description that we give of it.

A living system is a particular type of machine. Humberto Maturana, a Chilean biologist and epistemologist, together with Francisco Varela, developed the theory of autopoiesis. According to this theory, the understanding of how living systems are organized is possible only when they are considered as autoreferential units, that is self-organizing and self-producing entities. It is important to grasp the keywords that accompany the idea of autopoiesis: living systems refer to biology, organization refers to cybernetics, and unity refers to epistemology. Maturana's thinking encompasses all of these keywords. An autopoietic system is, first and foremost, a homeostat in the sense that the variable that is held constant is its own organization.

An autopoietic machine continually generates its own organization. Its operation consists of the production of its own components, and it does so in an endless turnover of components and perturbation compensations. An autopoietic machine is a circular and recursive system in which at each step the components necessary for the next step are generated. An excellent biological example of this circularity and recursion is given by cellular metabolism.

We can therefore divide the machines into two large classes: in one we put the autopoietic machines, or those that are autonomous, capable of self-maintaining their own organization and therefore their own existence, their own way of life; in the other we put the allopoietic machines, those which have something other than themselves as the product of their functioning and which are therefore not able to generate themselves, they are not autonomous. Living beings are autopoietic machines while human artifacts, such as computers, are allopoietic machines. It is quite evident that a formal system with its axioms and inference rules is nothing more than an allopoietic machine.

According to the distinction mentioned above, we can divide machines into two major classes: in one, we place autopoietic machines, which are autonomous and capable of self-maintaining their organization and therefore their existence, their living; in the other, we have allopoietic machines, which produce something different from themselves as a result of their functioning and are thus unable to self-generate or be autonomous. Living beings are autopoietic machines, while human artifacts, such as computers, are allopoietic machines. It is quite evident that a formal system with its axioms and rules of inference is nothing more than an allopoietic machine.

Before discussing how all of this is related to mathematics, physics, and their connections, I would like to spend a few lines on the thoughts of neuroscientist Antonio Damasio (1999; 1994), who has highlighted a close relationship between emotion and rationality. The theoretical framework in which he places this association is based on the idea that the organism's reactions to environmental changes are necessary to preserve vital conditions through the maintenance of homeostatic equilibria (Damasio, 1999: 133-145; 1994: 135). Emotion, in fact, is understood as a reaction of the organism that is directed towards the body, inducing it to adapt to changes necessary to maintain its functional equilibria within the limits that allow life. Damasio's work show that without the perception of emotions, it is difficult to discern a stable pattern within a multitude of facts. Although Maturana does not explicitly mention the role of emotion, things are similar.

Let's now return to the central theme of the connection between mathematics and physics. We have seen that knowledge is a biological phenomenon, and therefore what determines the cognitive domain of a living being is its self-referential recursive organization. Since no interaction is possible that is not prescribed by its organization, inductive inference, in every creative process, is a necessary function that emerges from the self-referential circular organization. In other words, inductive inference is a structural property of living organization and therefore of the process of thought.

The connection with mathematics can be found in the writings of Henri Poincaré. In the first chapter of the book "Science and Hypothesis," he shows very clearly that the foundation of every mathematical reasoning is the inductive inference. Let's take a closer look at this, specifically thinking to arithmetic. Suppose we can prove that a proposition is true for $n=1$, and furthermore, if it is true for n, then it is also true for $n+1$. From this, we can conclude that the proposition is true for every n. To delve deeper, we can break down induction into its components: the proposition is true for the number 1. Now, if it is true for 1, it is true for 2, therefore it is true for 2. If it is true for 2, it is true for 3, and therefore it is true for 3. And so on, infinitely. It is a sequence of syllogisms where the conclusion of one serves as the input for the next. This infinite and countable sequence of syllogisms is captured in the single formula "if true for n, then true for n+1." At this point, Poincaré makes a very interesting observation. If we were interested in verifying the truth of a proposition for a fixed n, we would simply need to trace the chain from 1 up to n. But in this manner, as useful as the result may be, it would not have the rank of a general theorem valid for every n; it could not be, as Poincaré says, an "object of science." In other words, recursion, or proof by induction, contains within it the passage from the finite to the infinite, without which "...there would be no science, for there would be nothing general." But why do we strongly feel (here, the term "feel" can be understood in Damasio's sense) the evidence of the principle of induction? This is Poincaré's answer " It is because it is only the affirmation of the power of the mind which knows it can conceive of the

indefinite repetition of the same act, when the act is once possible. … Mathematical induction - ie., proof by recurrence – is necessarily imposed on us, because it is only the affirmation of a property of the mind itself".

The conclusion of the coherence between recursion as a mathematical object and recursion as the foundation of the autopoiesis of livings, and therefore as a biological phenomenon, can have consequences that may be interesting for some and perhaps nonsensical for others. A first question could be this: would an alien biology produce the same mathematics? I do not know. However, I know that we have been unable to produce mathematics other than the one we know. And what can we say about physics? In the epistemology of Maturana, but also of Heinz von Foerster, Prigogine and others, the observer plays a fundamental role. Maturana says: everything said is said by someone, meaning that every process of knowledge is autopoietic, that is, it derives from the action of a subject operating in a given domain. All doing is knowing, all action is knowledge, and all knowledge is action. The separation between subject and object typical of classical science begins to creak under the weight of the shift of attention from the observed systems to the observing systems, meaning by this, as indeed quantum mechanics shows, that there is no observation that can disregard the observer. In this epistemological perspective there is no distinction of principle between physics and mathematics, they are the same thing. I conclude these observations by quoting Heisenberg. " .. it is no longer possible to talk about the behavior of the particle independently of the observation process. This has the consequence that the laws of nature, which we formulate mathematically in quantum mechanics, no longer speak of the elementary particles themselves but of the knowledge we have of them. The idea of the objective reality of elementary particles has therefore surprisingly dissolved, and not in the fog of some new, unclear or misunderstood idea of reality, but in the transparent clarity of a mathematics that no longer represents the behavior of the particle, but the our knowledge about this behavior." In other words, mathematics and physics are the same thing.

I don't want to abuse the space for this commentary on the interesting article by Biao Wu, so I stop here with a final note. I think I have given hints that with an allopoietic machine, a computer, we cannot access the mathematical continuum, and this is its main limit. This is not a negligible point because even our everyday actions, choices and so on would be impossible without the continuum. The mathematical continuum can be accessed with the thought, that is, with the autopoiesis of Maturana and the emotions of Damasio, which is a prerogative of the human being. It would be interesting if Biao Wu could write his view.