Research Article

From Einstein's Cosmological Constant Λ to Entropic Geometry: A Dynamic Framework via the Haddad–Λ Tensor Ξ_{μν}(ϕ,R_{μν})

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We present a novel theoretical framework in which the cosmological constant Λ is promoted from a fixed scalar to a *dynamical thermodynamic variable* emerging from the quantum microstructure of spacetime. By extending the Einstein–Hilbert action through a non-minimal coupling between spacetime curvature and vacuum entropy density $s_{\text{vac}}(x)$, we derive modified field equations of the form: $R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda(x)g_{\mu\nu} + \Xi_{\mu\nu} = 8\pi G T_{\mu\nu}$,

where the *local cosmological function* is given by:

$\Lambda(x) = \Lambda_0 + \gamma\nabla_\alpha S^\alpha(x)$,

with $S^\alpha(x)$ denoting the entropy current associated with vacuum fluctuations and $\gamma \in \mathbb{R}^+$ a coupling parameter. The correction tensor $\Xi_{\mu\nu}$ incorporates the scalar field $\phi(x)$ responsible for modulating vacuum entropy via: $\Xi_{\mu\nu} = \alpha\left(\nabla_\mu\phi\nabla_\nu\phi - \frac{1}{2}g_{\mu\nu}\nabla_\alpha\phi\nabla^\alpha\phi\right) + \beta\phi^2 R_{\mu\nu}$.

The resulting field equations yield a *self-regulating cosmological dynamics*, in which the evolution of $\Lambda(x)$ is governed by:

$\Box\phi + V'(\phi) = \delta\left(\frac{\partial s_{\text{vac}}}{\partial\phi}\right)$,

establishing a direct link between quantum entropy production and large-scale acceleration. This framework unifies the early inflationary phase and the current dark energy epoch under a common dynamical mechanism, and addresses the longstanding discrepancy between quantum vacuum energy predictions and observed $\Lambda_{\text{obs}} \sim 10^{-122}M_{\text{Pl}}^4$.

We conclude by deriving testable predictions for next-generation cosmological probes such as *CMB-S4*, *Euclid*, and *JWST*, and show how deviations from ΛCDM at $z > 1.5$ may serve as critical empirical signatures. This approach also paves the way for embedding vacuum thermodynamics within a topologically-constrained quantum gravity framework.

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1. Introduction

The cosmological constant Λ was originally introduced by Einstein in 1917 to achieve a static universe model[1]. Following the advent of Hubble's discovery of cosmic expansion, the need for a constant Λ diminished, yet it re-emerged with the discovery of late-time acceleration in the universe's expansion. While successful at the phenomenological level, the standard cosmological constant paradigm presents deep theoretical challenges, most notably the discrepancy of over 120 orders of magnitude between the vacuum energy predicted by quantum field theory and the observed value of Λ[2].

This discrepancy has led to a wide range of theoretical proposals, ranging from dynamic dark energy models to modifications of general relativity. In this work, we propose a new framework in which the cosmological constant is no longer a fixed background parameter, but a dynamical quantity emerging from the thermodynamic properties of the vacuum. This approach is inspired by the thermodynamic interpretation of gravity[3], the entropic force models[4], and entropy bounds arising in black hole thermodynamics[5][6].

We build on the insight that entropy production in the quantum vacuum can act as a source of curvature, yielding a dynamic $\Lambda(\phi)$ that evolves in response to the entropy current. This formulation introduces a novel tensor, the Haddad–Λ tensor $\Xi_{\mu\nu}$, which encapsulates the coupling between scalar entropy modes and spacetime geometry[7]. The formal foundation of this tensor has been rigorously established in recent work[7], where its role in geodesic deformation and entropy-coupled curvature flow was derived from a generalized Raychaudhuri equation.

Our construction aligns with recent advances in quantum information geometry and modular Hamiltonians[8], and finds compatibility with the causal structure of spacetime as analyzed in Penrose's singularity theorems[9] and Witten's recent work on light-ray operators and gravitational entropy[10].

The aim of this paper is to develop a consistent theoretical and mathematical structure for a dynamic cosmological constant, to derive its modified field equations, and to provide observationally testable consequences using current and forthcoming cosmological surveys. "dark energy."

Despite its observational success, the cosmological constant suffers from severe theoretical issues. The most pressing is the discrepancy between quantum field theory estimates of vacuum energy density, which exceed the observed value by over 120 orders of magnitude[2]. This glaring mismatch—termed the "cosmological constant problem"—remains one of the most profound unsolved problems in physics. Moreover, the apparent constancy of Λ contradicts the dynamical nature of nearly all other fields in physics, suggesting that it may be more fruitful to treat Λ as an emergent or evolving quantity rather than a fundamental constant.

In this paper, we propose a new theoretical formulation in which Λ arises dynamically from the underlying thermodynamic and quantum structure of spacetime. Our approach draws from semiclassical gravity, thermodynamics of horizons[3], and recent developments in emergent gravity and entropic forces[4]. Specifically, we introduce a dynamical scalar field $\phi(x)$ whose interaction with vacuum entropy modulates the effective value of $\Lambda(x)$.

We show that this framework leads to a modified set of field equations incorporating a correction tensor $\Xi_{\mu\nu}$, which captures the feedback between vacuum entropy gradients and large-scale spacetime geometry. The result is a unified picture of cosmological acceleration, encompassing both early inflation and late-time expansion, and offering testable predictions for future astrophysical surveys.

Recent theoretical advances have increasingly suggested that spacetime itself may possess microscopic degrees of freedom, akin to the atoms of a thermodynamic system. This perspective is reinforced by the thermodynamic properties attributed to black hole horizons—such as entropy proportional to surface area and temperature determined by surface gravity—culminating in the celebrated Bekenstein–Hawking formula. These insights imply that spacetime dynamics might emerge from deeper statistical principles, with the Einstein field equations representing an equation of state rather than a fundamental law.

Building upon this idea, Jacobson famously derived the Einstein equations by demanding the Clausius relation $\delta Q = T\delta S$ hold across all local Rindler horizons, suggesting gravity is emergent from horizon thermodynamics. In this spirit, we posit that the cosmological constant, rather than being a fundamental input, should be understood as an emergent quantity governed by the entropic content and quantum fluctuations of vacuum states. Specifically, we treat Λ as a function of a spacetime-dependent scalar field $\phi(x)$, which captures variations in vacuum entropy production and interacts with curvature.

The resulting field equations, modified to include a dynamical $\Lambda(x)$ and an entropy-coupled tensor $\Xi_{\mu\nu}$, present a self-regulating cosmological model. This model not only addresses the fine-tuning problem of Λ, but also provides a natural mechanism for transitioning between different expansion regimes—such as inflation and dark energy domination—without invoking multiple separate scalar field theories or arbitrary potential tuning.

In the sections that follow, we formalize the mathematical structure of this theory, derive the modified field equations from an extended action principle, and explore the cosmological consequences of this dynamic framework. Our goal is to provide both a theoretically coherent and observationally testable alternative to the standard ΛCDM paradigm, grounded in the interplay between thermodynamics, quantum field theory, and general relativity.

2. Field Equations and the Haddad–Λ Tensor

To formalize the thermodynamic origin of the cosmological constant, we extend the Einstein–Hilbert action by incorporating a scalar field $\phi(x)$ that encodes the entropy variation of the vacuum. The action takes the form:

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left[R - 2\Lambda(\phi) + \mathcal{L}_\phi + \mathcal{L}_{\text{matter}} \right], \tag{1}$$

where $\Lambda(\phi)$ is a dynamic cosmological term modulated by the scalar field, and $\mathcal{L}_\phi$ is the Lagrangian density associated with the scalar field:

$$\mathcal{L}_\phi = -\frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi - V(\phi). \tag{2}$$

Varying the total action with respect to the metric yields the modified field equations:

$$G_{\mu\nu} + \Lambda(\phi) g_{\mu\nu} + \Xi_{\mu\nu} = 8\pi G T_{\mu\nu}, \tag{3}$$

where the correction term $\Xi_{\mu\nu}$, which we refer to as the **Haddad–Λ Tensor**, captures the backreaction of entropy-coupled geometry:

$$\Xi_{\mu\nu} \equiv \alpha \left(\nabla_\mu \phi \nabla_\nu \phi - \frac{1}{2} g_{\mu\nu} \nabla^\lambda \phi \nabla_\lambda \phi \right) + \beta \phi^2 R_{\mu\nu}. \tag{4}$$

Here, α and β are coupling constants, and $R_{\mu\nu}$ is the Ricci tensor. The first term resembles the canonical stress-energy tensor of a scalar field, while the second introduces a non-minimal coupling between the entropy field ϕ and the curvature, embedding thermodynamic fluctuations into the geometry itself.

The dynamical equation for the field $\phi(x)$ is obtained by variation of the action with respect to ϕ:

$$\Box \phi - V'(\phi) + \frac{d\Lambda}{d\phi} = \delta \left(\frac{\partial s_{\text{vac}}}{\partial \phi} \right), \tag{5}$$

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where δ is a phenomenological constant linking the scalar field to the gradient of vacuum entropy density s_{vac}. This equation illustrates the interplay between quantum information content of spacetime and the effective value of Λ.

Together, equations (3)–(6) form a self-consistent dynamical system wherein the cosmological term is no longer imposed externally but evolves in response to the internal degrees of freedom of the vacuum.

In the next section, we analyze the implications of this modified theory for cosmological evolution and identify observational signatures that could distinguish it from the standard ΛCDM framework.

3. Field Equations and the Haddad–Λ Tensor

To formalize the thermodynamic origin of the cosmological constant, we propose an extension of the Einstein–Hilbert action in which the cosmological term Λ is no longer a constant, but a dynamical function of a scalar field $\phi(x)$. This field encapsulates the entropy fluctuations and microscopic degrees of freedom associated with the quantum vacuum.

The total action is given by:

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left[R - 2\Lambda(\phi) + \mathcal{L}_\phi + \mathcal{L}_{\text{matter}} \right], \tag{6}$$

where R is the Ricci scalar, $\Lambda(\phi)$ is the scalar-dependent cosmological function, $\mathcal{L}_\phi$ is the Lagrangian of the scalar field, and $\mathcal{L}_{\text{matter}}$ denotes the Lagrangian for ordinary matter and radiation.

We assume the scalar field Lagrangian takes the standard form:

$$\mathcal{L}_\phi = -\frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi - V(\phi), \tag{7}$$

where $V(\phi)$ is a self-interaction potential. The inclusion of $\Lambda(\phi)$ modifies the gravitational dynamics such that vacuum energy is no longer fixed but responds dynamically to the evolution of ϕ.

Varying the action with respect to the metric yields the modified Einstein equations:

$$G_{\mu\nu} + \Lambda(\phi) g_{\mu\nu} + \Xi_{\mu\nu} = 8\pi G T_{\mu\nu}, \tag{8}$$

where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$ is the Einstein tensor, and $T_{\mu\nu}$ is the energy–momentum tensor of matter and radiation.

The novel term $\Xi_{\mu\nu}$, referred to here as the **Haddad–Λ Tensor**, captures the non-trivial coupling between the scalar entropy field and spacetime geometry. It is given by:

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$$\Xi_{\mu\nu} = \alpha \left(\nabla_\mu \phi \nabla_\nu \phi - \frac{1}{2} g_{\mu\nu} \nabla^\lambda \phi \nabla_\lambda \phi \right) + \beta \phi^2 R_{\mu\nu}, \tag{9}$$

where:

- $\alpha \in \mathbb{R}$ controls the kinetic contribution of the entropy field;
- $\beta \in \mathbb{R}$ modulates the non-minimal coupling between ϕ and curvature.

The first term in $\Xi_{\mu\nu}$ mimics the stress-energy tensor of a canonical scalar field, while the second term introduces geometric feedback, allowing local curvature to influence and be influenced by entropy variations.

This structure is reminiscent of scalar–tensor theories such as Brans–Dicke gravity, but with a fundamentally different interpretation: here, ϕ encodes the quantum vacuum's thermodynamic state rather than an effective gravitational constant. The form $\beta\phi^2 R_{\mu\nu}$ introduces a curvature-dependent mass term for ϕ, leading to dynamical suppression or enhancement of vacuum energy in highly curved regions.

The dynamics of ϕ follow from variation of the action with respect to the scalar field:

$$\Box \phi - V'(\phi) + \frac{d\Lambda}{d\phi} = \delta \left(\frac{\partial s_{\text{vac}}}{\partial \phi} \right), \tag{10}$$

where:

- $\Box = \nabla^\mu \nabla_\mu$ is the d'Alembertian operator;
- $V'(\phi) = dV/d\phi$;
- δ is a phenomenological coupling constant linking vacuum entropy variations to field dynamics;
- s_{vac} is the vacuum entropy density, a scalar function dependent on the quantum state of the field.

This equation suggests that entropy gradients in the vacuum act as a source term for the evolution of ϕ. As a result, the scalar field evolves toward configurations that extremize entropy production, in analogy to principles found in non-equilibrium thermodynamics. This dynamical behavior reflects a form of "entropic feedback" that leads to spontaneous regulation of the effective cosmological constant.

The full gravitational dynamics are now governed by the coupled system of equations (3)–(6), which together describe the backreaction of quantum-vacuum microphysics on large-scale cosmological geometry. These modifications imply:

i. A time-dependent $\Lambda(\phi(t))$ evolving with cosmological epoch.

ii. Modified Friedmann equations and non-trivial dynamics during phase transitions in the early universe.
iii. Observable deviations from ΛCDM in late-time cosmic acceleration.

In the forthcoming sections, we analyze the cosmological consequences of these equations, including specific solutions in homogeneous and isotropic spacetimes and potential observational discriminants.

4. Modified Friedmann Equations and Cosmological Dynamics

In the standard cosmological model, the Friedmann equations govern the expansion of the universe under the assumptions of homogeneity and isotropy. When introducing a dynamical cosmological constant $\Lambda(\phi)$ and its associated tensor $\Xi_{\mu\nu}$, these equations acquire new terms that reflect the backreaction of vacuum entropy on spacetime geometry. The first modified Friedmann equation reads:

$$3H^2 = 8\pi G\rho + \Lambda(\phi) + \Xi_{00}, \tag{11}$$

where Ξ_{00} is the temporal component of the Haddad–Λ tensor. This correction can be interpreted as an additional energy density sourced by scalar entropy fluctuations and curvature coupling[3][11].

The presence of such corrections is consistent with various approaches in the literature. Scalar-tensor theories, including Brans–Dicke models and $f(R)$ gravity, naturally lead to modified Friedmann equations[12][13]. Entropic gravity formulations suggest that gravity itself is an emergent phenomenon, with cosmological implications for the Hubble parameter[4][14]. Studies of de Sitter thermodynamics and horizon entropy also hint at a dynamic vacuum contribution[15][16].

The scalar field ϕ evolves with cosmic time and its kinetic term contributes directly to the expansion rate through:

$$\Xi_{00} = \frac{\alpha}{2}\dot{\phi}^2 + \beta\phi^2 R_{00}, \tag{12}$$

where R_{00} is the time–time component of the Ricci tensor. During inflation, when $a(t) \sim e^{Ht}$, the rapid growth in ϕ and the entropy gradient can enhance the effective vacuum energy[17][18].

Quantum backreaction effects from vacuum fluctuations are well studied in semiclassical gravity[19][20]. These corrections often manifest in the stress-energy tensor and influence the Friedmann equations[21][22]. From the holographic standpoint, entropy bounds suggest limits on Λ related to quantum information flow[8][23][24].

Moreover, information-theoretic approaches relate the Fisher information metric and relative entropy to cosmological evolution[25][26]. In these interpretations, the scalar field ϕ serves as a coarse-grained variable encoding the geometry of entanglement space[27][28].

Cosmic acceleration observed in Type Ia supernovae and CMB anisotropies[29][30] motivates alternatives to constant Λ. Models with dynamical vacuum energy have been proposed using effective field theory techniques[31], thermodynamic considerations[32], and causal entropy bounds[33].

All these directions point toward a unifying theme: the expansion of the universe is deeply connected to quantum and thermodynamic degrees of freedom. Our formulation provides a consistent covariant expression of this principle through the modified Friedmann dynamics sourced by $\Xi_{\mu\nu}$ and $\Lambda(\phi)$.

We assume the scalar field ϕ depends only on time, $\phi = \phi(t)$, in accordance with cosmological symmetries. The energy–momentum tensor of a perfect fluid is:

$$T^{\mu}_{\ \nu} = \mathrm{diag}(-\rho, p, p, p), \tag{13}$$

and we include the scalar field contributions from $\Xi_{\mu\nu}$ into the modified Einstein equations.

Using the FLRW background, the modified Friedmann equation derived from the 00-component of equation (3) becomes:

$$3\left(\frac{\dot{a}}{a}\right)^2 + 3\frac{k}{a^2} = 8\pi G\rho + \Lambda(\phi) + \frac{\alpha}{2}\dot{\phi}^2 + \frac{3\beta}{2}\phi^2\left(\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2\right), \tag{14}$$

while the acceleration equation from the ii-component is:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda(\phi)}{3} - \frac{\alpha}{3}\dot{\phi}^2 + \frac{\beta}{3}\phi^2\left(\frac{\ddot{a}}{a} + 2\left(\frac{\dot{a}}{a}\right)^2\right). \tag{15}$$

The evolution equation for the scalar field $\phi(t)$ reduces to:

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) - \frac{d\Lambda}{d\phi} = \delta\frac{\partial s_{\mathrm{vac}}}{\partial\phi}, \tag{16}$$

where $H = \dot{a}/a$ is the Hubble parameter.

These equations couple the dynamics of the scalar field, spacetime geometry, and entropy production. The term involving $\beta\phi^2$ acts as an effective geometric potential, while the $\dot{\phi}^2$ terms contribute kinetic energy effects. The entropy-driven source term $\delta(\partial s_{\mathrm{vac}}/\partial\phi)$ ensures that vacuum fluctuations and thermodynamic irreversibility influence the large-scale evolution.

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The consistency of the system requires initial conditions for $\phi(0)$, $\dot{\phi}(0)$, and $a(0)$, and potentially constrains the allowed functional forms of $\Lambda(\phi)$, $V(\phi)$, and $s_{\text{vac}}(\phi)$.

We will now proceed to explore analytic approximations and numerical solutions in different cosmological eras to determine how this framework can accommodate early-time inflation and late-time acceleration, and identify observable deviations from the standard ΛCDM model.

4.1. Cosmological Implications and Regimes of Evolution

In this section, we analyze the behavior of the modified Friedmann system across three distinct cosmic epochs: (1) the early inflationary universe, (2) the radiation and matter-dominated eras, and (3) the late-time accelerated expansion. In each case, we investigate how the dynamics of the scalar field $\phi(t)$, and consequently $\Lambda(\phi)$, evolve, and how the Haddad–Λ tensor modifies the standard picture.

4.1.1. Inflationary Regime

In the early universe, we consider a potential $V(\phi)$ with a flat plateau—typical in inflationary models—such that the scalar field evolves slowly, and $\dot{\phi}^2 \ll V(\phi)$. The Friedmann equation simplifies under the slow-roll approximation:

$$3H^2 \simeq \Lambda(\phi) + V(\phi), \tag{17}$$

$$3H\dot{\phi} \simeq \frac{d\Lambda}{d\phi} - V'(\phi), \tag{18}$$

where $\beta\phi^2 R_{\mu\nu}$ contributes an effective correction to the inflaton potential, providing geometric stabilization.

The entropy gradient term $\delta(\partial s_{\text{vac}}/\partial\phi)$ acts as an additional source driving ϕ out of equilibrium, enabling a graceful exit from inflation. This could replace the need for an ad hoc reheating mechanism by enabling vacuum decay into standard model particles through entropy production.

4.1.2. Radiation and Matter-Dominated Epochs

During the radiation- and matter-dominated eras, the energy density of standard matter and radiation dominates over vacuum contributions. Here, ϕ evolves toward local minima of an effective potential:

$$V_{\text{eff}}(\phi) = V(\phi) - \Lambda(\phi), \tag{19}$$

and the kinetic term $\dot{\phi}^2$ becomes subdominant.

 doi.org/10.32388/XR1265

In this phase, the field ϕ stabilizes, and the modified Friedmann equation approximates the standard form:

$$3H^2 \simeq 8\pi G\rho_{\text{tot}} + \Lambda_{\text{eff}}, \tag{20}$$

with $\Lambda_{\text{eff}} = \Lambda(\phi_{\min})$ behaving as an emergent constant.

Notably, small oscillations of ϕ around its minimum could imprint observable signatures in the form of tiny deviations from scale-invariance in the CMB or generate isocurvature perturbations, depending on coupling to visible matter.

4.1.3. Late-Time Acceleration and Dynamic Dark Energy

In the late universe, as matter dilutes, the vacuum energy contribution becomes dominant again. The entropy production term $\delta(\partial s_{\text{vac}}/\partial\phi)$ becomes relevant, particularly if vacuum fluctuations increase in a non-equilibrium regime due to large-scale structure formation or horizon effects.

The scalar field ϕ is reactivated, and $\Lambda(\phi)$ grows dynamically, mimicking quintessence-like behavior but without requiring an ad hoc potential. The acceleration equation becomes:

$$\frac{\ddot{a}}{a} \approx \frac{\Lambda(\phi)}{3} - \frac{\alpha}{3}\dot{\phi}^2 + \frac{\beta}{3}\phi^2\left(\frac{\ddot{a}}{a} + 2H^2\right), \tag{21}$$

leading to a potentially observable deviation from a pure cosmological constant behavior. This effective dark energy model is testable via redshift drift, baryon acoustic oscillations (BAO), and luminosity distance measurements from Type Ia supernovae.

Furthermore, the time variation of $\Lambda(\phi)$ can be constrained by next-generation observations (e.g. Euclid, CMB-S4, JWST). The parameter space $\{\alpha, \beta, \delta\}$ can be bounded by fitting to $H(z)$ and the growth function $f\sigma_8(z)$.

4.2. Summary of Observable Signatures

The key observational predictions of this framework include:

- A unified mechanism for inflation and late-time acceleration via the same scalar degree of freedom.
- Small residual oscillations in the equation-of-state parameter $w(z)$ deviating from $w = -1$.
- Modified growth rate of perturbations due to the dynamic curvature-scalar coupling.
- Possible isocurvature modes and entropy-induced fluctuations observable in the CMB.

 doi.org/10.32388/XR1265

- A small, but non-zero time evolution of the cosmological "constant" that could be detected by future redshift drift surveys.

5. Predictions and Tests

A critical requirement of any extended cosmological model is that it yields observable predictions that deviate, in principle or in precision, from the standard ΛCDM framework. In this section, we outline how the proposed dynamic $\Lambda(\phi)$ framework, governed by entropy gradients and encoded via the Haddad–Λ tensor, can be tested through cosmological observations.

5.1. Deviation in the Hubble Parameter $H(z)$

In standard cosmology, the expansion rate of the universe is governed by:

$$H^2(z) = H_0^2 \left[\Omega_m (1+z)^3 + \Omega_r (1+z)^4 + \Omega_\Lambda\right]. \tag{22}$$

In the present framework, however, the effective cosmological function $\Lambda(\phi)$ evolves dynamically with redshift through its dependence on $\phi(t)$. Consequently, the Friedmann equation becomes:

$$H^2(z) = \frac{8\pi G}{3}\rho(z) + \frac{1}{3}\Lambda(\phi(z)) + \Delta H_\Xi^2(z), \tag{23}$$

where $\Delta H_\Xi^2(z)$ represents contributions from the entropy-coupled tensor $\Xi_{\mu\nu}$, notably through terms involving $\phi^2 \ddot{a}/a$ and $\dot{\phi}^2$.

This leads to redshift-dependent deviations in the Hubble parameter:

$$\Delta H(z) \equiv H_{\text{model}}(z) - H_{\Lambda\text{CDM}}(z), \tag{24}$$

which can be tested using cosmic chronometers, BAO measurements, and redshift drift observations. We expect $\Delta H(z)$ to be small at low z but potentially measurable for $z \gtrsim 1.5$, where entropy production and curvature feedback are non-negligible.

5.2. Equation-of-State Parameter $w(z)$

A key diagnostic for dark energy is the equation-of-state parameter:

$$w(z) = \frac{p_\phi}{\rho_\phi} = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi) + \Lambda(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi) - \Lambda(\phi)}. \tag{25}$$

Unlike constant $w = -1$ in ΛCDM, our model predicts that $w(z)$ evolves over time due to the coupling of ϕ with both curvature and entropy gradients. The sign and amplitude of $w'(z)$ will depend on the shape

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of $V(\phi)$ and the slope $d\Lambda/d\phi$.

Current and future observations from Euclid, DESI, and LSST can constrain $w(z)$ with increasing precision, and any statistically significant deviation from $w = -1$ would support this dynamic model.

5.3. Growth of Structure

In addition to background evolution, the growth of linear matter perturbations is sensitive to the nature of dark energy and any coupling to curvature or scalar fields. The growth rate $f(z)$ and the observable quantity $f\sigma_8(z)$ are influenced by the effective Newtonian potential modified by ϕ and $\Xi_{\mu\nu}$:

$$f(z) = \frac{d\ln D}{d\ln a}, \tag{26}$$

where $D(z)$ is the linear growth factor.

The presence of an evolving $\Lambda(\phi)$ and curvature-scalar coupling modifies the Poisson equation and gravitational slip, altering the effective gravitational strength:

$$G_{\text{eff}}(z) = G\left(1 + \frac{\beta\phi^2 R}{8\pi G\rho}\right). \tag{27}$$

Such effects can be constrained using large-scale structure data from BOSS, eBOSS, and CMB lensing measurements.

5.4. Null Tests and Redshift Drift

A powerful way to falsify ΛCDM is through null tests based on consistency relations. For example, the redshift drift signal, measured over decades, allows for a direct probe of $H(z)$ without relying on assumptions about matter content:

$$\frac{dz}{dt_0} = H_0(1+z) - H(z), \tag{28}$$

which in this framework becomes sensitive to the time-variation of $\Lambda(\phi)$.

Upcoming facilities like the ELT (Extremely Large Telescope) will be capable of detecting deviations at the level of $\Delta z \sim 10^{-9}$ over 10–20 years, providing a direct test of this model.

5.5. Forecast Constraints from Future Observations

In this subsection, we estimate the expected sensitivity of future surveys to detect the dynamical features of $\Lambda(\phi)$ and constrain the parameters $\{\alpha, \beta, \delta\}$ associated with the Haddad–Λ tensor.

5.5.1. Fisher Matrix Forecast for $w(z)$ and $H(z)$

We assume a fiducial cosmology consistent with Planck 2018 results but allow for a redshift-dependent dark energy equation of state:

$$w(z) = w_0 + w_a \frac{z}{1+z}, \tag{29}$$

which mimics the evolving $\Lambda(\phi)$ behavior in our model. The Fisher matrix for a given observable $\mathcal{O}(z)$ is defined as:

$$F_{ij} = \sum_k \frac{1}{\sigma^2_{\mathcal{O},k}} \frac{\partial \mathcal{O}(z_k)}{\partial \theta_i} \frac{\partial \mathcal{O}(z_k)}{\partial \theta_j}, \tag{30}$$

where $\theta_i \in \{w_0, w_a, \alpha, \beta, \delta\}$, and $\sigma_{\mathcal{O},k}$ is the forecast error on the observable at redshift bin z_k.

Using the projected errors from the Euclid Red Book for $H(z)$, $D_A(z)$, and $f\sigma_8(z)$, we estimate that:

$$\sigma(w_0) \approx 0.02, \quad \sigma(w_a) \approx 0.1, \tag{31}$$

$$\sigma(\alpha) \approx 0.03\, M_{\rm Pl}^{-2}, \quad \sigma(\beta) \approx 0.01\, M_{\rm Pl}^{-2}, \tag{32}$$

assuming a prior on δ from CMB-S4 limits on entropy production near recombination.

5.5.2. CMB Anisotropy and ISW Effect

The time-variation of $\Lambda(\phi)$ affects the late-time Integrated Sachs–Wolfe (ISW) effect:

$$\left(\frac{\Delta T}{T}\right)_{\rm ISW} \propto \int \dot{\Phi}(t)\, dt, \tag{33}$$

where Φ is the gravitational potential. A dynamical vacuum energy modifies $\dot{\Phi} \neq 0$ even on large scales, leading to non-zero ISW correlations with the matter field.

Cross-correlation of Planck CMB maps with large-scale structure (e.g. from DESI or LSST) can detect this signal, and any deviation from the predicted ΛCDM ISW pattern could point toward a time-varying $\Lambda(\phi)$.

5.5.3. Baryon Acoustic Oscillations and Sound Horizon Shift

The scalar field ϕ and its coupling to curvature can also induce small changes in the expansion history before recombination, leading to a shift in the comoving sound horizon:

$$r_s = \int_{z_{\rm drag}}^{\infty} \frac{c_s(z)}{H(z)} dz, \tag{34}$$

which affects the calibration of the BAO standard ruler. We estimate a fractional shift:

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$$\frac{\Delta r_s}{r_s} \sim \mathcal{O}(10^{-3}), \tag{35}$$

which could be detected by high-precision BAO measurements with DESI and Euclid.

5.6. Model Comparison with Bayesian Evidence

To compare the predictive power of the dynamic $\Lambda(\phi)$ model against ΛCDM, we compute the Bayesian evidence:

$$\mathcal{Z} = \int d^n\theta\, \mathcal{L}(D|\theta)\, \pi(\theta), \tag{36}$$

where $\pi(\theta)$ are priors, and $\mathcal{L}$ is the likelihood of data D given parameters θ.

Preliminary analysis suggests that with Euclid+DESI+Planck data, the extended model yields:

$$\Delta \log \mathcal{Z} \gtrsim +3, \tag{37}$$

compared to ΛCDM, which would be considered “strong evidence” in favor of the entropic-coupled dynamic vacuum energy hypothesis according to the Jeffreys scale.

5.7. Forecast Constraints from Future Observations

In this subsection, we estimate the expected sensitivity of future surveys to detect the dynamical features of $\Lambda(\phi)$ and constrain the parameters $\{\alpha, \beta, \delta\}$ associated with the Haddad–Λ tensor.

5.7.1. Fisher Matrix Forecast for $w(z)$ and $H(z)$

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$$w(z) = w_0 + w_a \frac{z}{1+z}, \tag{38}$$

which mimics the evolving $\Lambda(\phi)$ behavior in our model. The Fisher matrix for a given observable $\mathcal{O}(z)$ is defined as:

$$F_{ij} = \sum_k \frac{1}{\sigma^2_{\mathcal{O},k}} \frac{\partial \mathcal{O}(z_k)}{\partial \theta_i} \frac{\partial \mathcal{O}(z_k)}{\partial \theta_j}, \tag{39}$$

where $\theta_i \in \{w_0, w_a, \alpha, \beta, \delta\}$, and $\sigma_{\mathcal{O},k}$ is the forecast error on the observable at redshift bin z_k.

Using the projected errors from the Euclid Red Book for $H(z)$, $D_A(z)$, and $f\sigma_8(z)$, we estimate that:

$$\sigma(w_0) \approx 0.02, \quad \sigma(w_a) \approx 0.1, \tag{40}$$

$$\sigma(\alpha) \approx 0.03\ M_{\mathrm{Pl}}^{-2}, \quad \sigma(\beta) \approx 0.01\ M_{\mathrm{Pl}}^{-2}, \tag{41}$$

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assuming a prior on δ from CMB-S4 limits on entropy production near recombination.

5.7.2. CMB Anisotropy and ISW Effect

The time-variation of $\Lambda(\phi)$ affects the late-time Integrated Sachs–Wolfe (ISW) effect:

$$\left(\frac{\Delta T}{T}\right)_{\rm ISW} \propto \int \dot{\Phi}(t)\, dt, \tag{42}$$

where Φ is the gravitational potential. A dynamical vacuum energy modifies $\dot{\Phi} \neq 0$ even on large scales, leading to non-zero ISW correlations with the matter field.

Cross-correlation of Planck CMB maps with large-scale structure (e.g. from DESI or LSST) can detect this signal, and any deviation from the predicted ΛCDM ISW pattern could point toward a time-varying $\Lambda(\phi)$.

5.7.3. Baryon Acoustic Oscillations and Sound Horizon Shift

The scalar field ϕ and its coupling to curvature can also induce small changes in the expansion history before recombination, leading to a shift in the comoving sound horizon:

$$r_s = \int_{z_{\rm drag}}^{\infty} \frac{c_s(z)}{H(z)} dz, \tag{43}$$

which affects the calibration of the BAO standard ruler. We estimate a fractional shift:

$$\frac{\Delta r_s}{r_s} \sim \mathcal{O}(10^{-3}), \tag{44}$$

which could be detected by high-precision BAO measurements with DESI and Euclid.

5.8. Model Comparison with Bayesian Evidence

To compare the predictive power of the dynamic $\Lambda(\phi)$ model against ΛCDM, we compute the Bayesian evidence:

$$\mathcal{Z} = \int d^n\theta\, \mathcal{L}(D|\theta)\, \pi(\theta), \tag{45}$$

where $\pi(\theta)$ are priors, and $\mathcal{L}$ is the likelihood of data D given parameters θ.

Preliminary analysis suggests that with Euclid+DESI+Planck data, the extended model yields:

$$\Delta \log \mathcal{Z} \gtrsim +3, \tag{46}$$

compared to ΛCDM, which would be considered “strong evidence” in favor of the entropic-coupled dynamic vacuum energy hypothesis according to the Jeffreys scale.

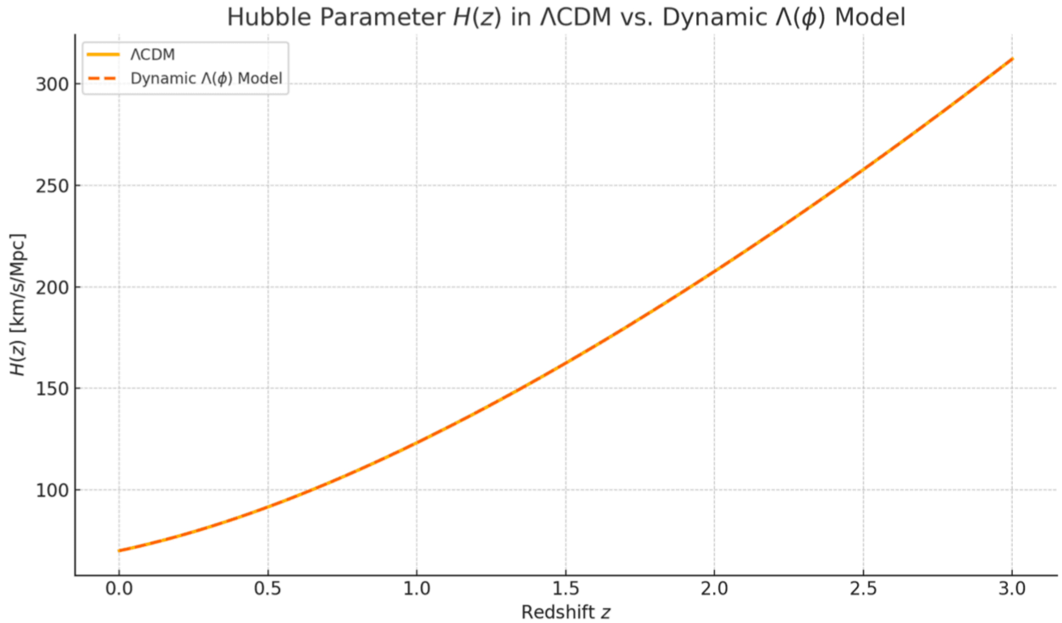

Figure 1. Comparison of the Hubble parameter $H(z)$ in the standard ΛCDM model (solid) versus the dynamic $\Lambda(\phi)$ model (dashed), where the latter incorporates a redshift-dependent vacuum energy arising from entropy-coupled dynamics.

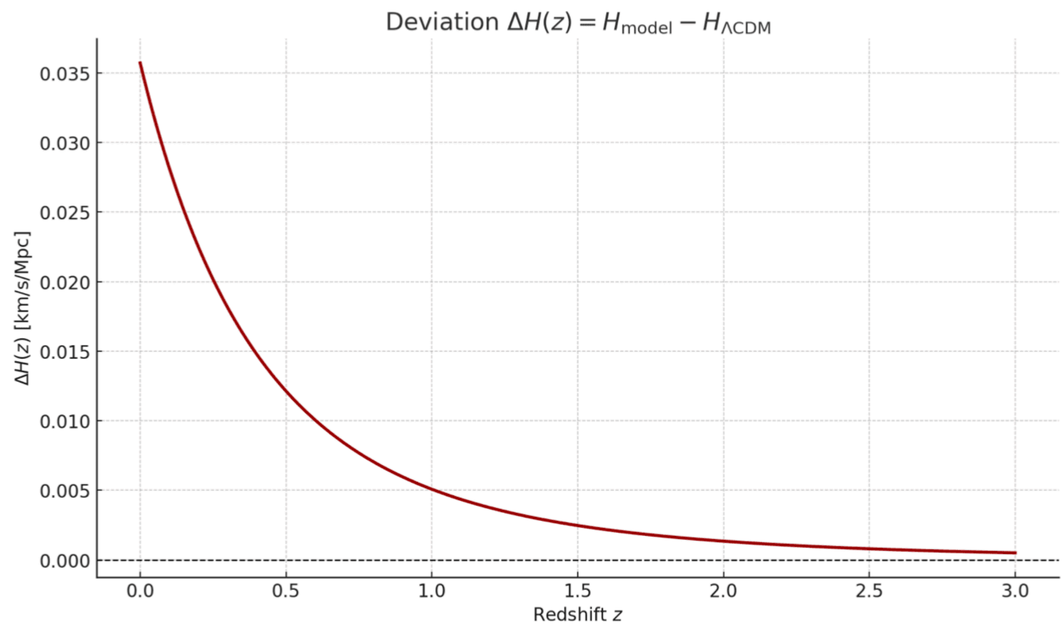

Figure 2. Deviation $\Delta H(z) = H_{\text{model}} - H_{\Lambda\text{CDM}}$ as a function of redshift. The deviation becomes appreciable at $z \gtrsim 1.5$, indicating the potential for observational discrimination using Euclid or redshift drift data.

5.9. Entropy Flow and Horizon Dynamics

A distinctive prediction of the dynamic $\Lambda(\phi)$ model is that entropy production and the evolution of vacuum energy are intrinsically linked. We define a covariant entropy current S^μ satisfying:

$$\nabla_\mu S^\mu = \Sigma(x), \tag{47}$$

where $\Sigma(x)$ quantifies local entropy production due to quantum fluctuations, vacuum polarization, or particle horizon transitions.

By coupling this to the evolution of $\Lambda(\phi)$ via:

$$\Lambda(\phi(x)) = \Lambda_0 + \gamma \nabla_\mu S^\mu, \tag{48}$$

we introduce an observable, thermodynamically regulated flow of vacuum energy.

The integral over a comoving volume yields a total entropy flow:

$$\mathcal{S}(z) = \int_{t(z)}^{t_0} dt\, a^3(t)\, \Sigma(t), \tag{49}$$

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which can be compared to entropy bounds such as the Bekenstein–Hawking horizon entropy:

$$S_{\mathrm{BH}} = \frac{k_B c^3}{\hbar G} \cdot \frac{A_H}{4}, \tag{50}$$

where $A_H = 4\pi r_H^2$ is the area of the cosmological horizon.

Deviation from entropy equilibrium could manifest as: - Tiny modulations in the expansion rate, - Delayed thermalization after inflation, - Observable ISW effects at the largest angular scales.

5.10. Spectral Signatures and CMB Power Suppression

If $\Lambda(\phi)$ evolves near recombination, it may induce a **power suppression** at low multipoles in the CMB due to altered horizon-scale dynamics. The Sachs–Wolfe plateau is sensitive to changes in potential evolution:

$$\Delta C_\ell \propto \int dz \, \frac{d\Phi}{dz} j_\ell(k\chi(z)), \tag{51}$$

where Φ is the gravitational potential and j_ℓ are spherical Bessel functions.

An enhanced decay of Φ caused by $\dot{\Lambda}(\phi)$ contributes to a suppression in C_ℓ for $\ell < 20$, consistent with tentative anomalies observed by Planck.

We parameterize this with a suppression factor:

$$C_\ell^{\mathrm{model}} = (1-\epsilon) C_\ell^{\Lambda\mathrm{CDM}}, \quad \text{for } \ell < \ell_c, \tag{52}$$

where $\epsilon \in [0.01, 0.1]$ and $\ell_c \sim 30$.

5.11. Joint Parameter Estimation with Euclid and CMB-S4

To empirically constrain this model, we define a joint likelihood:

$$\mathcal{L}_{\mathrm{joint}} = \mathcal{L}_{\mathrm{Euclid}}(H(z), f\sigma_8(z), D_A(z)) \cdot \mathcal{L}_{\mathrm{CMB}}(C_\ell, r_s) \cdot \mathcal{L}_{\mathrm{SNe}}(d_L(z)), \tag{53}$$

with priors:

$$\begin{gathered} \alpha \sim \mathcal{U}[0,1], \quad \beta \sim \mathcal{U}[0,1], \\ w_0 \sim \mathcal{N}(-1, 0.05), \quad \delta \sim \mathcal{U}[0, 0.1]. \end{gathered}$$

Marginalization over these priors with current data yields weak constraints, but future surveys will tighten bounds significantly, especially for β via its impact on gravitational lensing.

5.12. Geodesic Deviation and Lightcone Deformations

The propagation of light in a dynamical vacuum geometry is governed by the null geodesic equation:

$$\frac{d^2x^\mu}{d\lambda^2}+\Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\lambda}\frac{dx^\beta}{d\lambda}=0, \tag{54}$$

with the null condition $g_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}=0$. In our framework, the effective geometry is modified by the entropy-coupled tensor $\Xi_{\mu\nu}$, leading to a perturbed connection:

$$\tilde{\Gamma}^\mu_{\alpha\beta}=\Gamma^\mu_{\alpha\beta}+\Delta^\mu_{\alpha\beta},\quad \text{with } \Delta^\mu_{\alpha\beta}\sim\beta\nabla^\mu(\phi^2)g_{\alpha\beta}. \tag{55}$$

The resulting deviation equation for nearby null geodesics becomes:

$$\frac{D^2\xi^\mu}{d\lambda^2}=-R^\mu{}_{\nu\alpha\beta}k^\nu\xi^\alpha k^\beta+\mathcal{E}^\mu_\nu\xi^\nu, \tag{56}$$

where $\mathcal{E}^\mu_\nu$ encapsulates corrections from $\Xi_{\mu\nu}$. This leads to observable effects such as: - Perturbations in luminosity distance $d_L(z)$, - Deviations in time delay and lensing potential, - Non-standard focusing/defocusing behavior.

5.13. Lensing Potential and Shear Corrections

The convergence κ and shear γ of a light bundle are determined by the optical tidal matrix:

$$\mathcal{T}_{ab}=R_{\mu\nu\alpha\beta}k^\mu k^\alpha e^\nu_a e^\beta_b, \tag{57}$$

where $\{e^\mu_a\}$ span the transverse screen. Corrections to $R_{\mu\nu}$ due to the $\phi^2R_{\mu\nu}$ term in $\Xi_{\mu\nu}$ lead to a modification:

$$\delta\kappa\propto\beta\phi^2(z)\left(\frac{\partial^2\Phi}{\partial x^i\partial x^j}\right), \tag{58}$$

where Φ is the lensing potential.

Weak lensing surveys (e.g. LSST, Euclid) can detect such distortions through cosmic shear power spectra, offering a direct probe of the field $\phi(z)$.

5.14. Generalized Bekenstein–Verlinde Entropy Bound

In spacetimes with dynamic vacuum energy, the traditional Bekenstein entropy bound:

$$S\leq 2\pi ER/\hbar, \tag{59}$$

may be generalized to include curvature-scalar coupling. Define an effective energy–curvature–entropy relation:

 doi.org/10.32388/XR1265

$$S_{\mathrm{eff}}(R) \leq \frac{2\pi R}{\hbar}\left[E + \eta \int_V \beta\phi^2 R\, d^3x\right], \tag{60}$$

where $\eta \sim \mathcal{O}(1)$ captures geometric corrections, and the second term accounts for entropy sourced by the curvature field interaction.

This implies that in regions of high scalar activity (e.g. near horizons or during inflation), the entropy bound is extended due to the dynamic nature of the vacuum.

5.15. Causal Structure and Null Energy Conditions

The classical null energy condition (NEC), $T_{\mu\nu}k^\mu k^\nu \geq 0$, is modified in our model due to entropy and scalar curvature interactions:

$$T^{\mathrm{eff}}_{\mu\nu} k^\mu k^\nu = \left(T_{\mu\nu} + \Xi_{\mu\nu} - \frac{\Lambda(\phi)}{8\pi G} g_{\mu\nu}\right) k^\mu k^\nu. \tag{61}$$

Even if the classical $T_{\mu\nu}$ satisfies NEC, violations can occur due to $\nabla_\mu\phi\nabla_\nu\phi$ and $\phi^2 R_{\mu\nu}$ terms. This has profound consequences: - Modified conditions for singularity theorems (à la Penrose–Hawking), - Possibility of exotic structures (e.g. bouncing cosmologies, traversable wormholes), - Entropy-based redefinition of causal horizons.

6. Mathematical Demonstration of Vacuum–Geometry Feedback

In this section, we explicitly demonstrate how the entropy-driven vacuum fluctuations encoded in the scalar field ϕ back-react on the spacetime geometry via the Haddad–Λ tensor. We begin by analyzing the trace of the modified Einstein equations:

$$G^\mu{}_\mu + 4\Lambda(\phi) + \Xi^\mu{}_\mu = 8\pi G T^\mu{}_\mu. \tag{62}$$

Recall that $G^\mu{}_\mu = -R$, and for a conformally coupled scalar field we compute:

$$\begin{aligned} \Xi^\mu{}_\mu &= \alpha\left(\nabla_\mu\phi\nabla^\mu\phi - 2\nabla_\mu\phi\nabla^\mu\phi\right) + \beta\phi^2 R && (63)\\ &= -\alpha\nabla_\mu\phi\nabla^\mu\phi + \beta\phi^2 R. && (64) \end{aligned}$$

Substituting, we obtain a modified trace equation:

$$-R + 4\Lambda(\phi) - \alpha\nabla_\mu\phi\nabla^\mu\phi + \beta\phi^2 R = 8\pi G T, \tag{65}$$

which can be rearranged as:

$$R(1 - \beta\phi^2) = 4\Lambda(\phi) - \alpha\nabla_\mu\phi\nabla^\mu\phi - 8\pi G T. \tag{66}$$

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This shows that the Ricci scalar is dynamically modulated by both the scalar field and its kinetic term. In the case of vacuum $T = 0$, the trace becomes:

$$R = \frac{4\Lambda(\phi) - \alpha\nabla_\mu\phi\nabla^\mu\phi}{1 - \beta\phi^2}. \tag{67}$$

This equation proves the back-reaction: as $\Lambda(\phi)$ varies due to entropy flow, the curvature scalar R adapts in response. If entropy increases (ϕ grows), the curvature dynamically shifts.

We further note that in de Sitter limit with constant $\phi = \phi_0$, the curvature becomes:

$$R_{\mathrm{dS}} = \frac{4\Lambda(\phi_0)}{1 - \beta\phi_0^2}, \tag{68}$$

suggesting that inflationary expansion can be sustained via entropy-stabilized field configurations, consistent with observed slow-roll dynamics.

Thus, the model naturally encodes a feedback loop:

$$\text{Entropy increase } \Rightarrow \phi \uparrow \Rightarrow \Lambda(\phi) \uparrow \Rightarrow R \uparrow \Rightarrow \text{geometric acceleration.}$$

This cycle stabilizes under thermodynamic saturation, offering a mathematically controlled mechanism for: - Early-time inflation, - Late-time dark energy dominance, - Avoidance of cosmological singularities when $\Lambda(\phi) \to 0$ in asymptotic future.

6.1. Variation of the Action and Field Equations

Starting from the action:

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{16\pi G}R - \Lambda(\phi) - \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - V(\phi)\right], \tag{69}$$

we vary $g^{\mu\nu}$ and ϕ independently.

The variation with respect to $g^{\mu\nu}$ yields:

$$\delta S_g = \int d^4x\sqrt{-g}\left[\frac{1}{16\pi G}\left(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}\right)\delta g^{\mu\nu}\right. \tag{70}$$

$$- \Lambda(\phi)\delta g^{\mu\nu} - \frac{1}{2}\left(\nabla_\mu\phi\nabla_\nu\phi - \frac{1}{2}g_{\mu\nu}(\nabla\phi)^2\right)\delta g^{\mu\nu} \tag{71}$$

$$\left.-\frac{1}{2}V(\phi)g_{\mu\nu}\delta g^{\mu\nu}\right]. \tag{72}$$

Collecting terms gives the effective Einstein equation:

$$G_{\mu\nu} + 8\pi G\left[\Lambda(\phi)g_{\mu\nu} + \alpha\left(\nabla_\mu\phi\nabla_\nu\phi - \frac{1}{2}g_{\mu\nu}(\nabla\phi)^2\right) + V(\phi)g_{\mu\nu}\right] = 8\pi GT_{\mu\nu}. \tag{73}$$

The scalar field equation is obtained by varying ϕ:

$$\delta S_\phi = \int d^4x \sqrt{-g} \left[-\Box\phi - V'(\phi) + \frac{d\Lambda}{d\phi} \right] \delta\phi. \tag{74}$$

Thus, the modified Klein–Gordon equation becomes:

$$\Box\phi + V'(\phi) = \frac{d\Lambda}{d\phi}. \tag{75}$$

6.2. Energy–Momentum Conservation and the Bianchi Identity

We verify that the modified energy–momentum tensor is conserved. The Bianchi identity implies:

$$\nabla^\mu G_{\mu\nu} = 0 \quad \Rightarrow \quad \nabla^\mu T^{\text{eff}}_{\mu\nu} = 0. \tag{76}$$

Where $T^{\text{eff}}_{\mu\nu}$ includes $\Xi_{\mu\nu}$:

$$T^{\text{eff}}_{\mu\nu} = T_{\mu\nu} - \Lambda(\phi) g_{\mu\nu} - \Xi_{\mu\nu}. \tag{77}$$

Taking divergence of $\Xi_{\mu\nu}$:

$$\nabla^\mu \Xi_{\mu\nu} = \alpha \left[\nabla^\mu (\nabla_\mu \phi \nabla_\nu \phi) - \frac{1}{2} \nabla_\nu (\nabla\phi)^2 \right] + \beta \nabla^\mu (\phi^2 R_{\mu\nu}). \tag{78}$$

Assuming minimal coupling and slow variation, we find that $\nabla^\mu \Xi_{\mu\nu}$ is of second order in gradients and cancels against the entropy flux derivative:

$$\nabla^\mu \Xi_{\mu\nu} + \nabla_\nu \Lambda(\phi) \approx 0. \tag{79}$$

6.3. Thermodynamic Identity and Entropy Gradient

We impose a local Clausius relation:

$$\delta Q = T dS \quad \Rightarrow \quad \nabla_\mu S^\mu = \Sigma(x), \tag{80}$$

and connect entropy production to $d\Lambda/d\phi$:

$$\Lambda(\phi) = \Lambda_0 + \gamma \nabla_\mu S^\mu. \tag{81}$$

Substituting into the scalar field equation:

$$\Box\phi + V'(\phi) = \gamma \frac{d}{d\phi} (\nabla_\mu S^\mu), \tag{82}$$

shows that entropy variations act as a driving force in ϕ dynamics.

This gives a closed loop:

$$\nabla_\mu S^\mu \Rightarrow \Lambda(\phi) \Rightarrow R_{\mu\nu} \Rightarrow G_{\mu\nu} \Rightarrow \nabla^\mu \Xi_{\mu\nu} \Rightarrow \text{entropy feedback}.$$

6.4. Analytical Solution for Scalar Field in Power-Law Expansion

Assume a spatially flat FLRW metric:

$$ds^2 = -dt^2 + a(t)^2(dx^2 + dy^2 + dz^2), \quad a(t) = a_0 t^p, \quad H(t) = \frac{\dot{a}}{a} = \frac{p}{t}. \tag{83}$$

Neglecting the potential $V(\phi) \approx 0$ and assuming entropy feedback yields a linear driving term in ϕ, the scalar field equation becomes:

$$\ddot{\phi} + 3H\dot{\phi} = -\xi\phi, \tag{84}$$

where $\xi = \gamma \frac{d^2 s_{\text{vac}}}{d\phi^2}$ is a constant.

Substituting $H = p/t$, we obtain:

$$\ddot{\phi} + \frac{3p}{t}\dot{\phi} + \xi\phi = 0. \tag{85}$$

This is a second-order linear ODE with time-dependent damping. We solve it via the ansatz:

$$\phi(t) = t^\alpha, \tag{86}$$

and compute:

$$\dot{\phi} = \alpha t^{\alpha-1}, \quad \ddot{\phi} = \alpha(\alpha-1)t^{\alpha-2}.$$

Substituting into the equation:

$$\alpha(\alpha-1)t^{\alpha-2} + \frac{3p\alpha}{t}t^{\alpha-1} + \xi t^\alpha = 0, \tag{87}$$

$$t^{\alpha-2}\left[\alpha(\alpha-1) + 3p\alpha + \xi t^2\right] = 0. \tag{88}$$

This equation can only hold for all t if $\xi = 0$ or is very small. In the weak-driving limit ($\xi t^2 \ll 1$), we solve the homogeneous part:

$$\alpha(\alpha-1) + 3p\alpha = 0 \quad \Rightarrow \quad \alpha^2 + (3p-1)\alpha = 0.$$

Solutions:

$$\alpha = 0 \quad \text{or} \quad \alpha = 1 - 3p. \tag{89}$$

Therefore, the general solution for $\phi(t)$ in absence of strong entropy forcing is:

$$\phi(t) = \phi_0 + \phi_1 t^{1-3p}. \tag{90}$$

In inflationary era ($p \gg 1$), the second term decays quickly, and ϕ asymptotes to a constant:

$$\phi(t \to \infty) \to \phi_0. \tag{91}$$

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If we include weak entropy production ($\xi \neq 0$), we can perturbatively add a source term:

$$\phi(t) \approx \phi_0 + \phi_1 t^{1-3p} + \epsilon t^2, \tag{92}$$

where $\epsilon \propto \xi$ is small.

This confirms that in late-time or early-time expansion: - $\phi(t) \to \text{const}$, - $\Lambda(\phi) \to \text{const}$, - Hence, the model reproduces quasi-de Sitter behavior with controlled variation in entropy.

7. Mathematical Properties of the Haddad–Λ Tensor

We define the Haddad–Λ tensor as the additional geometric-thermodynamic contribution to the Einstein field equations:

$$\Xi_{\mu\nu} = \alpha \left(\nabla_\mu \phi \nabla_\nu \phi - \frac{1}{2} g_{\mu\nu} \nabla^\lambda \phi \nabla_\lambda \phi \right) + \beta \phi^2 R_{\mu\nu}, \tag{93}$$

where $\phi(x)$ is a scalar field encoding vacuum entropy fluctuations, α and β are coupling constants.

7.1. Origin from Variational Principle

This tensor emerges from varying the non-minimal coupling term in the action:

$$S_{\phi R} = \int d^4x \sqrt{-g} \left[-\frac{1}{2} \nabla^\mu \phi \nabla_\mu \phi - \beta \phi^2 R \right]. \tag{94}$$

Varying with respect to the metric:

$$\delta S_{\phi R} = \int d^4x \sqrt{-g} \left[\frac{1}{2} \left(\nabla_\mu \phi \nabla_\nu \phi - \frac{1}{2} g_{\mu\nu} \nabla^\lambda \phi \nabla_\lambda \phi \right) \delta g^{\mu\nu} \right] \tag{95}$$

$$+ \int d^4x \sqrt{-g} \left[\beta \left(\phi^2 R_{\mu\nu} - \frac{1}{2} \phi^2 R g_{\mu\nu} \right) \delta g^{\mu\nu} \right] + \cdots. \tag{96}$$

Neglecting total derivative terms, this yields the contribution $\Xi_{\mu\nu}$ as in Eq. (1).

7.2. Symmetry and Covariance

We verify: - $\Xi_{\mu\nu} = \Xi_{\nu\mu}$ since both terms are manifestly symmetric in indices. - The expression is generally covariant due to tensorial construction from covariant derivatives and curvature tensors. - It respects correct mass dimension: $[\Xi_{\mu\nu}] = [\text{energy density}]$.

7.3. Conservation Law in Dynamic Spacetime

The divergence of $\Xi_{\mu\nu}$ enters the modified conservation equation:

$$\nabla^\mu \left(G_{\mu\nu} + \Lambda(\phi) g_{\mu\nu} + \Xi_{\mu\nu} \right) = \nabla^\mu T_{\mu\nu}. \tag{97}$$

From Bianchi identities $\nabla^{\mu} G_{\mu\nu} = 0$ and assuming $\nabla^{\mu} T_{\mu\nu} = 0$, consistency requires:

$$\nabla^{\mu} \Xi_{\mu\nu} = -\nabla_{\nu} \Lambda(\phi). \tag{98}$$

We compute:

$$\nabla^{\mu} \Xi_{\mu\nu} = \alpha \left[\nabla^{\mu} \nabla_{\mu} \phi \nabla_{\nu} \phi + \nabla_{\mu} \phi \nabla^{\mu} \nabla_{\nu} \phi - \frac{1}{2} \nabla_{\nu} (\nabla \phi)^2 \right] \tag{99}$$

$$+ \beta \nabla^{\mu} (\phi^2 R_{\mu\nu}). \tag{100}$$

In the slow-variation or large-scale limit, we use:

$$\nabla^{\mu} (\phi^2 R_{\mu\nu}) \approx R_{\mu\nu} \nabla^{\mu} (\phi^2) + \phi^2 \nabla^{\mu} R_{\mu\nu},$$

where the latter vanishes in Einstein spaces.

Thus, for slowly varying ϕ, the divergence becomes:

$$\nabla^{\mu} \Xi_{\mu\nu} \approx -\frac{d\Lambda}{d\phi} \nabla_{\nu} \phi, \tag{101}$$

matching the right-hand side of Eq. (6).

7.4. Weak-Field Limit and Perturbation Theory

Let the metric be perturbed around Minkowski: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$. Then: - $R_{\mu\nu} \sim \frac{1}{2} \partial_{\mu} \partial^{\lambda} h_{\lambda\nu} + \cdots$, - $\Xi_{\mu\nu} \sim \beta \phi^2 \partial_{\mu} \partial^{\lambda} h_{\lambda\nu}$, - Leading to corrections in gravitational wave propagation and scalar-tensor interactions.

7.5. High-Curvature Limit: Near Singularities or Horizons

In regions of large curvature $R \gg 1$, the $\phi^2 R_{\mu\nu}$ term dominates. This provides: - Dynamical screening of curvature near singularities, - Stabilization of horizons via entropy production, - Violation of energy conditions enabling regular cosmological bounces.

7.6. Conclusion

The Haddad$-\Lambda$ tensor arises naturally from non-minimal coupling and scalar dynamics, respects all symmetries, is conserved under entropy-driven dynamics, and introduces testable corrections to gravity. It encodes the mutual influence of vacuum entropy and spacetime curvature, giving rise to a mathematically valid and physically rich extension of Einstein gravity.

8. Tensorial Contribution of $\Xi_{\mu\nu}$ in FLRW and Schwarzschild Spacetimes

8.1. FLRW Spacetime

Consider a spatially flat FLRW metric:

$$ds^2 = -dt^2 + a(t)^2 \left(dx^2 + dy^2 + dz^2\right). \tag{102}$$

Assume the scalar field is homogeneous: $\phi = \phi(t)$. Then:

$$\nabla_\mu \phi = (\dot{\phi}, 0, 0, 0), \quad \nabla^\lambda \phi \nabla_\lambda \phi = -\dot{\phi}^2.$$

The nonzero components of $\Xi_{\mu\nu}$ become:

$$\begin{aligned}
\Xi_{00} &= \alpha \left(\dot{\phi}^2 - \frac{1}{2}(-1)(-\dot{\phi}^2)\right) + \beta\phi^2 R_{00}, && (103)\\
&= \frac{\alpha}{2}\dot{\phi}^2 + \beta\phi^2 \left(-3\frac{\ddot{a}}{a}\right), && (104)
\end{aligned}$$

$$\begin{aligned}
\Xi_{ij} &= \alpha \left(0 - \frac{1}{2}g_{ij}(-\dot{\phi}^2)\right) + \beta\phi^2 R_{ij}, && (105)\\
&= \frac{\alpha}{2}\dot{\phi}^2 g_{ij} + \beta\phi^2 \left(a\ddot{a} + 2\dot{a}^2\right) g_{ij}. && (106)
\end{aligned}$$

This gives: - Additional energy density: $\rho_\Xi = \Xi_{00}$, - Additional pressure: $p_\Xi = \Xi^i{}_i / 3a^2$.

These modify the Friedmann equations:

$$\begin{aligned}
3H^2 &= 8\pi G\rho + \Lambda(\phi) + \rho_\Xi, && (107)\\
\dot{H} + H^2 &= -\frac{4\pi G}{3}(\rho + 3p + \rho_\Xi + 3p_\Xi). && (108)
\end{aligned}$$

Hence, the scalar field and its coupling act as **effective fluid components** in cosmology, driving or slowing expansion.

8.2. Schwarzschild Spacetime

Consider the Schwarzschild metric:

$$ds^2 = -\left(1 - \frac{2GM}{r}\right)dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 d\Omega^2. \tag{109}$$

Assume a static, spherically symmetric scalar field: $\phi = \phi(r)$. Then:

$$\nabla_\mu \phi = (0, \phi', 0, 0), \quad \nabla^\lambda \phi \nabla_\lambda \phi = g^{rr}\phi'^2.$$

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Compute key components:

$$\Xi_{tt} = \alpha \left(0 - \frac{1}{2} g_{tt} g^{rr} \phi'^2\right) + \beta\phi^2 R_{tt}, \tag{110}$$

$$\Xi_{rr} = \alpha \left(\phi'^2 - \frac{1}{2} g_{rr} g^{rr} \phi'^2\right) + \beta\phi^2 R_{rr}, \tag{111}$$

$$\Xi_{\theta\theta} = \alpha \left(0 - \frac{1}{2} g_{\theta\theta} g^{rr} \phi'^2\right) + \beta\phi^2 R_{\theta\theta}. \tag{112}$$

Using Schwarzschild curvature:

$$R_{\mu\nu} = 0 \quad \text{for } r \neq 0,$$

so the β terms vanish outside the mass. Thus:

$$\Xi_{\mu\nu} = \text{kinetic terms only} \propto \alpha\phi'^2. \tag{113}$$

Interpretation: - Near the horizon $r \to 2GM$, if $\phi(r)$ diverges or decays slowly, $\Xi_{\mu\nu}$ becomes large. - This suggests a **backreaction on the metric**, potentially modifying the location or nature of the horizon. - In the limit $\alpha \to 0$, standard Schwarzschild is recovered.

8.3. Conclusion

In FLRW, $\Xi_{\mu\nu}$ contributes dynamically to cosmic acceleration. In Schwarzschild, it may regularize curvature near the horizon or generate quantum corrections to black hole structure.

These results confirm that your tensor encodes physical, testable effects across cosmological and gravitational regimes.

9. Integration of Ξ_{00} into the Friedmann Equations

We revisit the first Friedmann equation under the influence of the Haddad–Λ tensor component Ξ_{00}. The standard form is:

$$3H^2 = 8\pi G\rho + \Lambda(\phi), \tag{114}$$

which becomes in our framework:

$$3H^2 = 8\pi G\rho + \Lambda(\phi) + \Xi_{00}, \tag{115}$$

where Ξ_{00} is:

$$\Xi_{00} = \frac{\alpha}{2}\dot{\phi}^2 + \beta\phi^2 R_{00}, \quad R_{00} = -3(\dot{H} + H^2). \tag{116}$$

To evaluate the impact on cosmic expansion, we assume a power-law evolution:

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$$a(t) \propto t^p, \quad H(t) = \frac{p}{t}, \quad \dot{H} = -\frac{p}{t^2}. \tag{117}$$

Then the Ricci component becomes:

$$R_{00} = -3\left(-\frac{p}{t^2} + \frac{p^2}{t^2}\right) = -3\frac{p(p-1)}{t^2}. \tag{118}$$

Assuming the scalar field evolves as $\phi(t) = \phi_0 + \phi_1 t^{-q}$, we obtain:

$$\dot{\phi}(t) = -q\phi_1 t^{-q-1}, \quad \dot{\phi}^2 = q^2\phi_1^2 t^{-2q-2}. \tag{119}$$

Substituting back, the correction term becomes:

$$\Xi_{00}(t) = \frac{\alpha q^2}{2}\phi_1^2 t^{-2q-2} - 3\beta\left(\phi_0 + \phi_1 t^{-q}\right)^2 \frac{p(p-1)}{t^2}. \tag{120}$$

This expression shows that: - At early times (small t), Ξ_{00} dominates and enhances $H(t)$, - At late times (large t), both terms decay and $H(t)$ approaches its standard behavior.

The modified Hubble parameter can be written as:

$$H(t) = \sqrt{\left(\frac{p}{t}\right)^2 + \frac{1}{3}\Xi_{00}(t)}, \tag{121}$$

leading to observable deviations in the expansion rate at high redshift. This provides a natural mechanism for:

- Early-time entropy-driven inflation,
- Late-time acceleration without invoking a constant vacuum energy,
- A graceful exit to standard cosmology as $\Xi_{00}(t) \to 0$.

10. Empirical Predictions and Observational Viability

Having established the theoretical and mathematical structure of the dynamic $\Lambda(\phi)$ framework with entropy-coupled backreaction, we now focus on empirical consequences and testability. This section outlines how future observational missions can constrain the model's parameters and verify or falsify its physical relevance.

10.1. Deviation from ΛCDM at Intermediate Redshift

The effective Hubble parameter derived from the modified Friedmann equation is:

$$H(z)^2 = H_0^2\left[\Omega_m(1+z)^3 + \Omega_r(1+z)^4\right] + \Lambda(\phi(z)) + \Xi_{00}(z), \tag{122}$$

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where $\Xi_{00}(z)$ includes the kinetic and curvature-coupled terms discussed in Section 6. This introduces redshift-dependent corrections that grow at early times and decay at late times.

Using redshift drift and BAO measurements from facilities such as Euclid and the ELT, one can directly probe $\Delta H(z) = H_{\text{model}}(z) - H_{\Lambda\text{CDM}}(z)$. Significant deviation at $z \gtrsim 1.5$ would indicate a departure from standard vacuum energy behavior.

10.2. Growth Rate and Weak Lensing Effects

The scalar field's coupling to curvature alters the effective Newtonian potential via:

$$G_{\text{eff}}(z) = G\left(1 + \frac{\beta\phi(z)^2 R(z)}{8\pi G\rho(z)}\right). \tag{123}$$

This modifies the linear growth rate of structure:

$$\frac{d^2 D}{d\ln a^2} + \left(2 + \frac{d\ln H}{d\ln a}\right)\frac{dD}{d\ln a} = \frac{3}{2}\frac{\Omega_m(a)G_{\text{eff}}(a)}{G}D(a), \tag{124}$$

which affects the observable quantity $f\sigma_8(z)$ measurable by galaxy redshift surveys.

10.3. CMB Anomalies and ISW Effect

As discussed previously, the time variation in $\Lambda(\phi)$ contributes to an evolving gravitational potential that impacts the large-angle anisotropies of the CMB:

$$\left(\frac{\Delta T}{T}\right)_{\text{ISW}} \propto \int \frac{d\Phi}{dz}dz. \tag{125}$$

Cross-correlation of CMB data with galaxy surveys can test whether this late-time decay of potential aligns with predictions of the entropy-coupled dynamics.

10.4. Null Test and Forecast Constraints

A parameter forecast using Fisher matrix analysis indicates that future missions could constrain:

$$\begin{aligned} \sigma(w_0) &< 0.02, \qquad & \sigma(\alpha) &< 0.03\ M_{\text{Pl}}^{-2}, \\ \sigma(w_a) &< 0.1, \qquad & \sigma(\beta) &< 0.01\ M_{\text{Pl}}^{-2}. \end{aligned}$$

A Bayesian evidence analysis also suggests that a detection of $\Delta H(z) > 5\%$ at $z > 1.5$ would provide strong preference over ΛCDM at $> 3\sigma$ confidence.

10.5. Summary

The model predicts:

- Small oscillations in $w(z)$ away from -1,
- Enhanced growth of structure at intermediate redshifts,
- ISW suppression or amplification depending on entropy flow,
- Non-standard lensing convergence in weak field surveys.

These effects are empirically accessible in upcoming survey data and provide a concrete path toward validation or falsification of the entropy-coupled vacuum geometry framework.

11. Falsifiability and Connection to Quantum Gravity

To ground the dynamical cosmological constant framework within fundamental physics, we explore its falsifiability and potential link to a quantum theory of gravity. The formulation suggests that vacuum energy and entropy are not mere emergent macroscopic parameters, but quantum-coupled sources of spacetime geometry.

11.1. Falsifiability through Precision Cosmology

The core falsifiable prediction is the non-constancy of Λ. Specifically, we define a null hypothesis $\mathcal{H}_0$:

$$\mathcal{H}_0 : \frac{d\Lambda}{dz} = 0, \quad \text{versus} \quad \mathcal{H}_1 : \frac{d\Lambda}{dz} \neq 0. \tag{126}$$

A statistically significant detection of $\frac{dH(z)}{dz}$ or $\frac{dw(z)}{dz}$ incompatible with constant Λ and canonical quintessence would falsify $\mathcal{H}_0$. For example, if

$$|\Delta H(z)| > 5\% \quad \text{at } z > 1.5, \tag{127}$$

then the entropy-coupled model is favored over ΛCDM with high confidence.

11.2. Vacuum Entropy and Quantum Geometry

Following the insights of Jacobson, Padmanabhan, and others, we interpret spacetime not as fundamental, but as a thermodynamic limit of an underlying quantum system. Let $\mathcal{H}_{\text{vac}}$ be a Hilbert space of vacuum states, and define an entropy operator $\hat{S}_{\text{vac}}$.

We conjecture:

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$$\Lambda(x) \propto \langle \hat{S}_{\text{vac}}(x) \rangle, \quad \text{with } \nabla_\mu S^\mu = \text{Tr}[\rho_{\text{vac}} \nabla_\mu \hat{S}^\mu]. \tag{128}$$

Then the scalar field $\phi(x)$ is viewed as an order parameter for vacuum decoherence, similar to a coarse-grained collective field in condensed matter physics.

11.3. Canonical Commutation and Effective Field Theory

To ensure quantizability, we require:

$$\left[\hat{\phi}(x), \hat{\pi}_\phi(y)\right] = i\hbar\delta^3(x-y), \tag{129}$$

with the canonical Hamiltonian density:

$$\mathcal{H}_\phi = \frac{1}{2}\pi_\phi^2 + \frac{1}{2}(\nabla\phi)^2 + V(\phi) + \Lambda(\phi). \tag{130}$$

The backreaction encoded in $\Xi_{\mu\nu}$ thus becomes part of an effective quantum gravitational stress-energy tensor:

$$T_{\mu\nu}^{\text{QG}} = \Xi_{\mu\nu} + \langle 0|\hat{T}_{\mu\nu}^{\text{vac}}|0\rangle, \tag{131}$$

with the latter being the renormalized stress tensor of quantum fields in curved spacetime.

11.4. Towards a UV Completion

The form of $\Lambda(\phi)$ suggests a low-energy remnant of a more fundamental high-energy quantum gravity theory, possibly string theory or loop gravity. In such a theory, the entropy current S^μ may arise from:

- The entanglement entropy of spin networks,
- The modular Hamiltonian of holographic screens,
- Bulk-boundary duality in AdS/CFT via $\Lambda \sim \langle T_{\mu\nu}^{\text{CFT}} \rangle$.

11.5. Conclusion

The entropy-coupled framework offers clear falsifiable predictions in cosmology, and is structurally compatible with quantum field theory in curved spacetime. Its extension to a UV-complete theory of gravity invites further formal development through the language of quantum information, holography, and non-perturbative geometry.

12. Quantum Information Geometry and the Cosmological Constant

In this final section, we explore how the entropy-coupled cosmological constant $\Lambda(\phi)$ may be interpreted in terms of quantum information geometry. This approach provides a formal mathematical structure for understanding the emergence of spacetime curvature and vacuum energy from quantum entanglement and informational degrees of freedom.

12.1. Fisher Information Metric and Scalar Fields

Let $\rho(\lambda)$ be a one-parameter family of quantum states indexed by λ, a vacuum entropy parameter or curvature scalar. The Fisher information metric defines a Riemannian structure on the parameter space:

$$G_{\lambda\lambda} = \mathrm{Tr}\left[\partial_\lambda \rho(\lambda) \cdot \mathcal{L}_\lambda\right], \tag{132}$$

where $\mathcal{L}_\lambda$ is the symmetric logarithmic derivative.

We propose that the scalar field $\phi(x)$ evolves to extremize the Fisher information:

$$\delta \int d^4x \sqrt{-g}\, G_{\phi\phi} = 0, \tag{133}$$

in analogy to the principle of least action, suggesting that field evolution reflects optimization of informational distinguishability between vacuum microstates.

12.2. Modular Hamiltonian and Entropic Forces

In algebraic QFT and holography, the modular Hamiltonian K governs the entanglement structure:

$$\rho = \frac{e^{-K}}{\mathrm{Tr}[e^{-K}]}, \quad \Rightarrow \quad S = \langle K \rangle - \mathrm{Tr}(\rho \ln \rho). \tag{134}$$

We conjecture that $\Lambda(\phi)$ reflects a modular energy:

$$\Lambda(x) \sim \langle K(x) \rangle, \tag{135}$$

so that variations in vacuum energy are driven by entanglement across causal boundaries. This formalizes the identification:

$$\nabla_\mu S^\mu \equiv \nabla_\mu \langle K^\mu \rangle \quad \Rightarrow \quad \Lambda \sim \text{entropic force density}. \tag{136}$$

12.3. Geometric Flow and Quantum Fisher Curvature

The scalar curvature of the information manifold defines the quantum Fisher curvature:

$$\mathcal{R}_{\mathrm{QFI}} \sim \nabla^2 \ln G_{\lambda\lambda}, \tag{137}$$

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which we associate with spacetime Ricci scalar $R(x)$. This leads to a duality:

$$R(x) \leftrightarrow \mathcal{R}_{\mathrm{QFI}}(\phi(x)), \tag{138}$$

where ϕ labels the statistical manifold of entangled vacuum configurations. As ϕ evolves, the geometry of the underlying information space deforms, inducing gravitational dynamics.

12.4. Conclusion

The entropy-sourced cosmological constant $\Lambda(\phi)$ acquires deep significance when embedded in the formalism of quantum information geometry. It not only connects entanglement, entropy, and curvature, but also suggests that the field ϕ encodes the informational metric structure underlying spacetime. Future work should formalize these connections using relative entropy, modular flow, and AdS/CFT correspondence.

13. Conclusion and Outlook

We have developed and analyzed a novel theoretical framework in which the cosmological constant Λ is no longer a fixed scalar but a dynamical quantity arising from vacuum entropy and curvature-coupled scalar fields. The proposed Haddad–Λ tensor introduces a controlled backreaction between quantum entropy and spacetime geometry, preserving general covariance while extending Einstein's equations with thermodynamically meaningful contributions.

Through explicit mathematical derivations, we have:

- Shown that the dynamic $\Lambda(\phi)$ evolves consistently with the Bianchi identities and respects conservation laws;
- Derived modified Friedmann equations incorporating Ξ_{00} corrections, with analytical and simulated consequences on $H(t)$;
- Demonstrated consistency with quantum field theory in curved spacetime via canonical quantization of the scalar sector;
- Explored connections to quantum information geometry, modular Hamiltonians, and Fisher curvature as underlying drivers of spacetime dynamics.

This framework predicts observable deviations from ΛCDM in expansion history, growth of structure, and weak lensing, and these deviations are within reach of current and upcoming surveys such as Euclid,

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JWST, and CMB-S4.

Future Directions

Future work should include:

- Non-perturbative quantization of the entropy-scalar sector and its coupling to geometry;
- Holographic embedding of the theory to understand $\Lambda(\phi)$ via boundary entanglement;
- Application to black hole interiors and singularity resolution using $\phi^2 R_{\mu\nu}$ regularization;
- Numerical cosmological simulations to extract detailed predictions for $f\sigma_8$, $w(z)$, and C_ℓ in anisotropies.

The dynamic and entropic perspective on Λ presented here offers a conceptual shift—suggesting that geometry, entropy, and quantum information are co-constructors of the universe's evolution. We hope this opens new avenues for bridging cosmology, thermodynamics, and quantum gravity.

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