Review of: "A Random Journey Through the Math of Gambling"

Maurice Clerc

Potential competing interests: No potential competing interests to declare.

Unnecessarily complicated.
The examples are not new. Why not, but you should at least propose explanations and proofs that are simpler than the ones easily available elsewhere.

You provide only eight references.
Of course you can't be exhaustive but more are needed, particularly the ones that proposes simpler proofs.

Just browse the Net with the keywords "random walk along a line" or "one-dimensional random walk" or even "simple random walk" that is the usual name.
And then "biased random walk".

You can even request a "coin flip random walk." You will observe that it is a typical homework assignment for students and some sites propose a solution.

You could also search for "two dimension random walk".

Please clarify the intended audience for your paper.
You write
"It is designed for anyone who wants to get a clearer idea of the mechanisms that govern gambling and games of chance in their mathematical implications, which, as paradoxical, also become psychological."

However, I don't believe it to be true. A novice or someone with limited experience may become discouraged, as they can easily access simpler explanations and proofs.
And an expert will not learn anything.

Introduction
----
p. 2
"the probability of winning at each step is only 5% below the 50% which is typical of a fair game"

Difficult to agree. The claim suggests that the probability of winning at each step is 45%, which is 5% below the 50% expected in a fair game. If the game is fair, each outcome should have an equal chance of occurring, resulting in a 50%

probability of winning. Therefore, the claim is not correct. If the probability of winning at each step is consistently 45%, the game would be biased against the player.

The Ballot Problem

----

p. 3

"A good way to list the votes for each candidate, at each step n of the counting process, is as follows. We can mark on a blackboard a +1 when the vote is assigned
to P and a 1 when the vote is assigned to Q."

A good way?
In the Abstract you write:
"Much emphasis is placed on concrete examples"
Is this a concrete example?
I am not aware of examples of this counting method (instead of using two increasing counters) for real elections.
Please give references.

p. 4
"This lemma provides the probability that, if P is leading at step n, then it has always led at all previous steps."

This is true only because the unrealistic formula (1). Not very useful then.
It is not uncommon, when tallying votes, to see candidate P initially being a loser and eventually ending up winning. This can even happen multiple times during the counting process.

The Ballot theorem does not assume your counting method.
Moreover, one can easily find several proofs, even on Wikipedia, including some far simpler than yours:
https://en.wikipedia.org/wiki/Bertrand%27s_ballot_theorem

So I don't think that this example is very useful, as it doesn't seem realistic and is unnecessarily complicated.

Tossing coins

----

p. 6
"The probability of the event E is then P(E) = 2^(N-2) /2^N = 25%"

The usual and far simpler approach to the event E = {two heads at the first two trials} is to just multiply the probabilities: (1/2)*(1/2)=1/4.
More generally, most of your results can be obtained by simply applying classic combinations of probabilities.

The Ruin problem

Gambler's ruin is also a classic problem. Even in Wikipedia, you can find it and its relationship with coin flipping (biased or not).

And, again, on the Net, just try the request "gambler's ruin random walk".

See, for instance,

https://web.mit.edu/neboat/Public/6.042/randomwalks.pdf

Conclusion

"We first observed that, in a fair game,"

As seen on page 2, your definition of a fair game is not correct.

"we proved that in an unfair game, although it may seem weird, the event related to a return to the origin is transient, that is, it will occur only a finite number of times even if we repeat the trials ad infinitum."

See

https://www.math.ucla.edu/~biskup/PDFs/PCMI/PCMI-notes-1

Appendix

You are duplicating certain proofs that have already been presented in published works. Even if some of these papers are included in the list of references, you should cite them explicitly in the paper. Moreover, your approach is at times more complex.

See, for example,

[3] Y. Kochetkov, An easy proof of Polya's Theorem on Random Walks