

Review of: "The smallest gap between primes"

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This result would be great if correct, but it doesn't seem to be. In particular, the claim before the line "which is trivially true for a sufficiently large prime number p_n " is in fact false for sufficiently large p_n . In particular,

$$\prod_{p_k \geq p_n} p_k^2/(p_k^2-1) \leq \prod_{m \geq p_n} (1-O(1/m^2)) = \exp(O(1/m)),$$

which approaches 1 as n goes to infinity. In particular, for sufficiently large n ,

$$2(1 - \prod_{p_k \geq p_n} p_k^2/(p_k^2-1))$$

is arbitrarily close to 0, and thus LESS than (rather than bigger than) $\log(\zeta(2))$.

Furthermore, it seems very unlikely that any proof technique along these lines can be made to work. None of the kinds of inequalities that are used here would be violated if the prime numbers were essentially evenly spaced (i.e. if the gap between the n th and $n+1$ st prime was always approximately $\log(n)$). Therefore, inequalities of this form alone should be insufficient to prove the existence of small prime gaps.