Review of: "New Approximate Symmetry Theorems and Comparisons With Exact Symmetries"

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Potential competing interests: No potential competing interests to declare.

In this article, the author introduces three new theories of approximate symmetry. These approximate symmetries are contrasted with each other and with exact symmetries. The theories are applied to nonlinear ordinary differential equations for which exact solutions are available. It is shown that from the symmetries, approximate solutions, as well as exact solutions in some restricted cases, can be retrieved. Depending on the specific approximate theory and the equations considered, the approximate symmetries may expand the Lie algebra of the exact symmetries, may be a perturbed form of the exact symmetries, or may be a subalgebra of the exact symmetries. Both exact and approximate solutions are retrieved using these symmetries. The authors have structured the work in 5 sections, demonstrating meticulous organization. Their work represents a significant contribution to approximation theory. After a thorough review, I have verified the correctness of all the solution procedures presented. The results presented in this paper are promising for applications in Analytic Number Theory and related fields. Therefore, the following recommendation is proposed: 1. Carefully review all the equations that have labels in your work and remove the labels that you are not going to use. For example, in equation (2.1), remove the label. 2. Check carefully all the work; it has many punctuation errors. 3. Align all equations. In recent years, special functions have emerged as key players in the field of approximation theory. Their growing importance is attributed to their versatility in representing and solving a wide range of problems in theoretical and applied mathematics. Therefore, I recommend citing the following works to delve deeper into their applications and theoretical foundations: 1. Quintana, Y., Ram´ırez, W., and Urieles, A.: On an operational matrix method based on generalized Bernoulli polynomials of level m. Calcolo. 55(3) (2018), 29 pages. https://doi.org/10.1007/s10092- 018-0272-5. 2. Cesarano, C., Ram´ırez, W., Diaz, S.: New results for degenerated generalized Apostol–Bernoulli, Apostol–Euler and Apostol–Genocchi polynomials. WSEAS Transactions on Mathematics. 2022, 21 604-608. 3. G., Dattoli, H.M, Srivastava, C., Cesarano.: Laguerre and Legendre polynomials from an operational point of view. Applied Mathematics and Computation. Volume 124, Issue 1, 10 November 2001, Pages 117-127 4. Cesarano, C.: Identities and generating functions on Chebyshev polynomials. Georgian Mathematical Journal, 2012, 19(3), pp. 427–440. 5. Cesarano C.; Assante, Dario.: A note on generalized Bessel functions. International Journal of Mathematical Models and Methods in Applied Sciences. 7(6), (2013), pp. 625-629 6. C. Cesarano, Indentities and generating functions on Chebyshev polynomials, Int. J. of Pure Mathematics, 19 (2012), 427–440. 7. C. Cesarano, Generalizations of two-variable Chebyshev and Gegenbauer polynomials, Int. J. of Applied Mathematics & Statistics (IJAMAS), 53 (2015), 1–7. 8. G., Dattoli, P. E. Ricci and C., Cesarano.: The Lagrange Polynomials, the Associated Generalizations, and the Umbral Calculus. Integral Transforms and Special Functions.14:2, (2013) 181-186 https://doi.org/10.1080/1065246031000098186

I recommend its publication.