



Representations and Implications of Papers Written by E.T. Whittaker in 1903 and 1904

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Abstract. In 1903 and 1904, E.T. Whittaker produced a pair of papers that, although sometimes considered mere mathematical statements (Barrett 1993), held important implications if viewed as physical realities. His 1903 paper united electrostatic and gravitational attraction as both resulting from longitudinal waves in the medium. His 1904 paper specifically dealt with electromagnetic waves as resulting from the interference of two such longitudinal waves as scalar potential functions. These papers, if representative of physical reality, may have important implications: gravitational lensing, gravitational waves, the Aharonov-Bohm effect, the existence of a hyperspace above or behind normal space, and the elimination of gravitational and point charge singularities.

Keywords: Whittaker, Gravitational Lensing, Gravitational Waves, Aharonov-Bohm Effect, Singularities

Introduction

In light of the fact that gravitational theories today face several difficulties – the requirement for dark energy, the requirement for dark matter, the existence of unphysical gravitational singularities, and incompatibility with Quantum Mechanics – it may be useful to uncover and explore older classical theories of gravity that overtly or implicitly offered several features of General Relativity. One such theory was put forward by E.T. Whittaker in 1903 and 1904. His 1903 paper on partial differential equations anticipated General Relativity in many ways by positing both an undulatory theory of gravity and a static gravitational field that resulted from effects propagating through the field. His 1904 paper on two scalar potential functions constructed the electromagnetic wave from two scalar potential functions, showing that the electromagnetic four-potential of Relativity may possess a degree of arbitrariness. This paper will serve to review these two overlooked papers and highlight some of their overt and implicit consequences.

Representations of the Works of E.T. Whittaker

E.T. Whittaker's 1903 paper "On the Partial Differential Equations of Mathematical Physics" found a general harmonic solution to the Wave Equation and the more specific Laplace Equation in three dimensions in such a way that both potentials could be analyzed into simple plane waves. An electrostatic or gravitational field, varying with the inverse square of distance, results from such propagating effects. The general solution to the Laplace equation was in the form:

$$\int_0^{2\pi} f(x\cos v + y\sin v + iz, v)dv \quad (1)$$

This is achieved by expanding the function f as a Taylor series with respect to the first argument and a Fourier series with respect to the second argument. The v is considered a periodic argument. Using similar techniques, the general solution to the Wave Equation was found in the form:

$$\int_0^\pi \int_0^{2\pi} f(x\sin u \cos v + y\sin u \sin v + z\cos u + ct, u, v)dudv \quad (2)$$

Whittaker considered "gravitation and electrostatic attraction explained as modes of wave-disturbance" (Whittaker 1903). Once these longitudinal waves interfere with each other and an action is set up, the disturbance at any point does not depend on time but only on position; they are standing waves. Therefore, the force potential can be defined in terms of both standing waves (global, or non-local solution) and by propagating waves (local solution changing in time) (Barrett 1993). This produced an undulatory theory of gravity propagating with a finite velocity and subsumed gravity to the transmission of electromagnetic radiation. That the aether produced longitudinal as well as transverse electromagnetic waves was a common belief of 19th century physicists and was given a mathematically rigorous treatment by Whittaker in 1903 (Carvalho and Rodrigues 2008).

Hector Munera writes that Whittaker's claim of generality is unacceptable due to the assumption of periodicity. Authors subsequent to Whittaker wrote that, in most practical cases, the solution is expected to vary harmonically in time. Munera also importantly reminds the reader that equation (1) implies a rotation in the complex plane ($z, ix\cos\theta + iysin\theta$). Finally, Munera writes that a time-dependent solution in the form of equation (2) is arrived at by "projecting z and $x\cos\theta + y\sin\theta$ onto ray r directed at angle ϕ relative to the Z -axis, thus shifting to spherical coordinates" (Munera 2018).

In 1904, Whittaker's paper "On an Expression of the Electromagnetic Field due to Electrons by Means of Two Scalar Potential Functions" showed that EM fields specifically could be decomposed into two scalar potential functions that

could be assembled as intersecting beams in which ordinary EM waves appear. Whittaker achieved this by first defining three scalar fields which are related by time and space derivatives to the traditional scalar electric potential and vector magnetic potential. One of the fields is eliminated by choosing a gauge. The two scalar potentials F and G can derive the magnetic force h and dielectric displacement d as:

$$\begin{aligned}
d_x &= \frac{\partial^2 F}{\partial x \partial z} + \frac{1}{c} \frac{\partial^2 G}{\partial y \partial t} \\
d_y &= \frac{\partial^2 F}{\partial y \partial z} - \frac{1}{c} \frac{\partial^2 G}{\partial x \partial t} \\
d_z &= \frac{\partial^2 F}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 G}{\partial t^2} \\
h_x &= \frac{1}{c} \frac{\partial^2 F}{\partial y \partial t} - \frac{\partial^2 G}{\partial x \partial z} \\
h_y &= -\frac{1}{c} \frac{\partial^2 F}{\partial x \partial t} - \frac{\partial^2 G}{\partial y \partial z} \\
h_z &= \frac{\partial^2 G}{\partial x^2} + \frac{\partial^2 G}{\partial y^2}
\end{aligned} \tag{3}$$

F and G are represented in terms of magnitude as:

$$\begin{aligned}
F(x, y, z, t) &= \sum \frac{e}{4\pi} \sinh^{-1} \frac{\bar{z}' - z}{((\bar{x}' - x)^2 + (\bar{y}' - y)^2)^{1/2}} \\
G(x, y, z, t) &= \sum \frac{e}{4\pi} \tan^{-1} \frac{\bar{y}' - y}{\bar{x}' - x}
\end{aligned} \tag{4}$$

The summation is taken over all the electrons in the field. In continuous form they are represented as:

$$\begin{aligned}
F &= \int_0^\pi \int_0^{2\pi} f(x \sin u \cos v + y \sin u \sin v + z \cos u + ct, u, v) du dv \\
G &= \int_0^\pi \int_0^{2\pi} g(x \sin u \cos v + y \sin u \sin v + z \cos u + ct, u, v) du dv
\end{aligned} \tag{5}$$

Whittaker rewrote these two scalar potentials in his 1951 version of A History of the Theories of Aether and Electricity as:

$$F(x, y, z, t) = \frac{1}{2} \sum_e \log \frac{\bar{r} + \bar{z}' - z}{\bar{r} - (\bar{z}' - z)}$$

$$G(x, y, z, t) = -i \frac{1}{2} \sum_e \log \frac{\bar{x}' - x + i(\bar{y}' - y)}{\bar{x}' - x - i(\bar{y}' - y)} \quad (6)$$

Whittaker wrote:

It will be noted that F and G are defined in terms of the positions of the electrons alone, and do not explicitly involve their velocities. Since in the above formulae for d and h an interchange of electric and magnetic quantities corresponds to a change of G into F and of F into G, it is clear that the two functions F and G exhibit the duality which is characteristic of electromagnetic theory: thus an electrostatic field can be described by F alone, and a magnetostatic field by G alone; again, if the field consists of a plane wave of light, then the functions F and G correspond respectively to two plane-polarised components into which it can be resolved. Since there are an infinite number of ways of resolving a plane wave of light into two plane-polarised components, it is natural to expect that, corresponding to any given electromagnetic field, there should be an infinite number of pairs of functions F and G capable of describing it, their difference from each other depending on the choice of the axes of co-ordinates-as is in fact the case. *Thus there is a physical reason why any particular pair of functions F and G should be specially related to one co-ordinate, and cannot be described by formulae symmetrically related to the three co-ordinates (x,y,z)* (Whittaker 1951).

Implications of the Works of E.T. Whittaker

The combination of Whittaker's 1903 paper on gravitation and electrostatic attraction and his 1904 paper on electromagnetic wave propagation shed new light on these phenomena in three dimensions by treating the z direction (propagation direction) differently than the other two spatial directions. Gravity may thus manifest itself solely in the y direction (Yurth 2007) due to the preferred direction of the potentials and the mass-proportionality of the medium with respect to propagation speed (Laszlo 2003). Gravitational lensing would result from the two scalar potentials interfering with each other to create electromagnetic waves by means of their mass-proportionality and preferred direction.

These papers account not only for gravity and electromagnetism as modes of

disturbance in the same medium, but could provide an explanation for gravitational waves. Whittaker explicitly claimed that his theory of gravitational and electromagnetic potentials provided for an “undulatory theory of gravity” (Whittaker 1903).

These papers directly anticipated the Aharonov-Bohm effect since the potentials F and G are considered the most basic entities. While fields require the potentials to exist, the potentials can exist on their own. This is demonstrated experimentally with the Aharonov-Bohm effect.

Additionally, the scalar potential (normally dimensionless) is given a discrete and quantized structure as resulting from the interference of two superluminal longitudinal waves in the medium itself. This medium is implied as hyperdimensional with its existence defined in terms of frequency resonance.

Singularities can potentially be eliminated by this theory. A point charge as one type of singularity would not exist; rather, charge would work collectively as longitudinal motion in the aether that carries electromagnetic radiation. The need for a propagation medium for transverse Hertzian waves was in fact seen by Maxwell and other 19th century physicists since the waves consist of orthogonal electric (E) and magnetic (B or H) waves. The E waves are undulating dipolar electric fields and all such dipoles were considered to require separated and opposite electric charges in the 19th century. The concept of massless charge as motion in the two scalar potentials would account for this. Furthermore, gravitational singularities would not need to exist. Solutions to light propagation around black holes were provided by Whittaker by first considering Maxwell’s equations in a dielectric medium (Whittaker 1928). Black holes can be viewed in the same light as the two scalar potentials, although they are complex and three-dimensional. Thus, black holes may work collectively as charge does, exist as part of the hyperspatial structure, and may even generate the Whittaker potentials (localized around each galaxy’s supermassive black hole) by extracting an aspect of mass from infalling matter.

Conclusion

The works of E.T. Whittaker have been neglected by the existing community of physicists, but may hold fruitful avenues for exploration. It is possible that a model of gravity based on these papers could do away with singularities as well as dark matter and dark energy. They could be developed and modelled in order to answer some of the open questions in physics.

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