

Peer Review

# Review of: "Inequality Emerges from Networks"

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## Review: DeCanio, Watkins (2025) Inequality Emerges from Networks

The paper "Inequality Emerges from Networks" studies how the structure of firm hierarchies evolves in a setting where firms have to perform two different tasks that favour different network structures. One task is an 'addition task' that imposes some processing cost on the nodes (i.e., employees) that increases with the number of in-links and, hence, limits the number of links a node will have. The other task is an 'adoption task' where innovation spreads through the network (lowering the node's processing cost) that doesn't incur any processing cost and, hence, would favour high-degree nodes. The firm hierarchies are supposed to perform well on both tasks, and they are optimized to perform well at this task using an evolutionary algorithm. The authors find that the resulting hierarchy produces inequality in compensation levels that is comparable to levels of income inequality that are reported in the literature.

I read this paper with great interest, as it proposes an interesting question and investigates an interesting trade-off problem. However, I have serious doubts about some of the implementation and, consequently, the results. I strongly encourage the authors to address my concerns, especially with respect to the network-optimization algorithm and the theoretical considerations behind the chosen parameter ranges.

I give the paper two stars for the interesting problem that it discusses, but because the results are not trustworthy until my concerns are addressed, I cannot rate it higher.

General comments:

- I don't think the network evolution mechanism appropriately covers the whole combinatorial space of possible networks. To me, it seems that the algorithm has a strong bias to stay close to the network

that was most efficient upon initialization. Minimizing non-linear functions of complex networks typically involves a rugged potential landscape with many local minima. Typically, researchers avoid getting stuck in local minima by applying some Metropolis-Hastings type algorithm. I would at least like to see some convergence studies that show that the algorithm actually works. Here's an example of a paper I recently came across, tackling a similar network optimization process: <https://arxiv.org/abs/2504.12955>

- The paper's model has -- at its core -- the trade-off between two mechanisms that favour different network structures. The tables show different parameter values for the mechanisms, but the trade-off is not discussed explicitly. I would like to see a phase diagram for the resulting network where the x-y axes are the (relative?) importance of the associative vs. the adoptive task, and the z-axis/color plot shows some value characterizing the network structure (e.g., the cost or Gini values discussed in the paper). This would allow us to understand the transition from a purely hierarchical network to a star-shaped network, and also show that the evolution algorithm produces the correct network structures in both edge cases!

#### Specific comments:

- If I understand Table 1 (and Tab. 2) correctly, they report the results of the Gini values of a single simulation run. However, for a stochastic optimization problem on random networks, we need to discuss standard errors/variances, etc., here! Re-run the analysis 500 times and report the mean Gini and some confidence intervals. The authors hint that they did this (by varying the initial seed) but don't report the results.
- Table 1 reports Gini coefficients between .27 and .70 for the optimized networks. It's no wonder that the empirical values fall within this arguably very large range. It would be great to have some theoretical considerations that limit the size of the parameter space. For example: What does it *mean* that the exponent is 2 or 5 for the respective cost functions? What *relative* importance does the adoption cost function have for a given denominator, and how close are we to the 'hierarchical' model of no adoption task?
- If you initialise your networks "randomly," you should specify what type of random network model you chose. (I guess Erdős-Renyi, but it doesn't say.) Why did you choose the network model? If your optimization is sensitive to the choice of the seed network, this has grave consequences!
- I would also expect a more thorough discussion of which employee compensation scheme makes the most sense. There's empirical data for this, or even just common sense: In a firm that is structured

hierarchically like the one in Fig. 1, we'd expect node #1 to get the highest compensation. However, it has Betweenness 0! In my humble opinion, this disqualifies Betweenness as a compensation mechanism. Some more theoretical considerations in this regard would be appropriate.

- One proposition on how to improve the network evolution algorithm: One of my concerns is that the most efficient firm is just copied many times, and the final result is a set of N copies of the same network. This could be avoided by evolving each separately and seeing where that takes the firm population. One could argue that this is also similar to what happens in reality: A firm optimises its structure without copying the entire structure of competing firms.

The simplest test to show that the optimization algorithm works is to apply it without the addition/adoption task and show that it produces the theoretically optimal network structures.

- I'd suggest a more specific title, e.g., "Inequality *in employee compensation* emerges from networks"

## Declarations

**Potential competing interests:** No potential competing interests to declare.