

Review of: "SAT is as hard as solving Homogeneous Diophantine Equation of Degree Two"

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In the article, "Canonical problems for quadratic programming and projective method for their solution", B. Kalantari, in Contemporary Mathematics, 114, 1990, the following problem is shown to be NP-complete: Given a symmetric integer matrix A , is $x^T A x = 0$ for some nontrivial $x \geq 0$. This is shown by reducing the knapsack problem to the above decision problem for some such matrix A satisfying the property that if a nontrivial nonnegative zero exists all its components are either zero or the same positive constant c . Hence after dividing by c we get a 0-1 solution. This results seem to cover the present author's results on Homogeneous Diophantine Equation. The author should consider them seriously. Nevertheless it is interesting that the author has rediscovered it. The significance of homogeneous programming is perhaps ignored in the literature in optimization for this reason I provide some additional comments below.

The significance of this homogeneous quadratic problem in Kalantari's article lies in the fact that linear programming over rational inputs can be reduced to it for a case where A is symmetric PSD. Moreover, in this case, exclusively, either such zero exists or there exists a positive definite diagonal matrix D such that $D A D e = e$, e the vector of ones. Diagonal scaling also exists in nonconvex homogeneous case but the duality is not exclusive. This matrix scaling duality for PSD case of A in turn gave rise to a simple polynomial-time diagonal scaling-linear programming by Khachiyan and Kalantari, SIAM, J. of Optimization, 1992. Generalizations and applications of homogeneous programming and matrix scaling to SDP and self-concordant homogeneous programming are also given by Kalantari.

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