Review of: "Multiplicity of solutions for nonlocal fractional equations with nonsmooth potentials"

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Potential competing interests: No potential competing interests to declare.

Questions, comments and suggestions to authors

In the sequel, we will present some questions and recommendations.

1. Please, in whole manuscript, when to cite a bibliography reference, refer the complete reference, e.g., [1, Chapter 2, Section 3] or [2, Lemma 3.2]. This citation style makes it easier for the reader to search such a reference.
2. Please, in whole manuscript, Standardize notation for dimension.
3. Page 01, The functions F, G and H are defined from where to where, they are incomplete in the text.
4. Page 01, It is not clear what is written in the sentence: “We denote by the generalized gradient of ∂F(x, u), ∂G(x, u) and ∂H(x, u) to u.” I suggest clarifying.
5. Page 02, In (1.3), what did the authors prove? Note that it is written in sequence, but the context of equation (1.3) is strange. I suggest removing the part “proved the following fractional Laplacian equation:”
6. Page 03, when defining space X, I suggest writing it as a set.
7. Page 03, Insert a geometric sketch of set O.
8. Page 05, please, Clarify the text after definition 2.2, it is very confusing. One question: Wouldn't this text be the definition of the generalized gradient of I(u)?
9. Page 05, please, Define duality notation in the generalized gradient definition.
10. Page 06, What does the notation 2^X* mean?
11. Please, in whole manuscript review the commands for Inf, Sup and limits indexes to actually stay in the index.
12. Page 06, After the definitions of H and H, the following text is completely disconnected from the rest of the text.
13. Page 07, In the conditions about F, write \xi(x,u) instead of only \xi. The same for the conditions about G and H.
14. Page 09, Throughout the page, check the commands to index the upper and lower limits. I suggest using \displaystyle\lim_{n \to \infty}.
15. Page 09, In the item (ii), Fatou's lemma is being used, not Fadou's lemma.
16. Page 09 and Page 08, I suggest writing the relationship between the functionals \Phi(u), \Psi(u) and \tilde{H}(u) and the functions \xi(x,u), \zeta(x,u) and eta(x,u).
17. Page 09, In the proof of Lemma 3.2, mention the immersion theorems that are being used.
18. Page 10, In the equation following (3.6), cite the immersion theorems again. The same after (3.7).

19. Page 11, In Theorem 3.1, please write what it means for u to be a critical point of \mu I(u) + \Psi(u) + \lambda \Phi(u) + \nu \tilde{H}(u).
20. Page 11, I suggest dividing the proof of Theorem 3.1 into two statements, namely: limitation and the behavior in (3.10).
21. Page 12, At the end of page 12, who is \hat{k}? Where does he appear in the following accounts?
22. Please, organize the entire reference; it is completely out of order. I suggest using alphabetical order.

Conclusion

The manuscript presents a very relevant problem that envolve nonsmooth potentials, which involves technical abstract (A critical point Theorem for nonsmooth functionals) and combining it with the analytical framework on fractional Sobolev spaces. Certainly these results will be useful in many lines of research in nonlinear theory of elliptic PDEs concerning in this área.

The presentation of manuscript It's not clear in some parts, I suggest it should be improved. Furthermore, it presents many inaccuracies in writing and organization. I suggest a wide reading to eliminate these inaccuracies. But, however, the mathematics present in the manuscript is very rich and important.

In conclusion, taking into account the previous comments, and after making the proposed suggestions, I believe that such a manuscript is a great and perfect work with regard to Functional Theory.