Decay Characteristics of Neutron Excess Phosphorous Nuclei

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Abstract

In neutron star mergers, neutron excess nuclei and the r-process are important factors governing the production of heavier nuclear systems. A single-particle model evaluation of phosphorous nuclei suggests that the heaviest Z = 15 nucleus will have mass 54 with filling of the $1f_{5/2}$ neutron shell. A = 45 – 54 phosphorous isotopes have limited experimental half-life data, but the model predicts beta decay half-lives in the range of 0.404 – 10.1 ms. Based on previous calculations for Z = 9 -14, 20, 26, and 30 systems and comparisons to the ^{45}P, ^{46}P, ^{47}P, and ^{49}P calculations, summarized in the Japanese Nuclear Data Compilation, the single-particle model results likely overestimate the half-lives of A = 45 – 54 neutron excess phosphorous nuclei.

1.0 Introduction

The nucleosynthesis of heavy elements occurs by three basic processes that add protons or neutrons to a nuclear system[1,2]. The p-process adds protons and the s- or slow process and r- or rapid process adds neutrons. Capture of protons by nuclear systems produces predominantly proton-rich nuclei that tend to decay by positron emission and electron capture[1,2]. Neutron capture creates neutron-rich nuclei, and the resulting nuclear systems depend upon the rate of neutron addition and the beta decay rates of the residual nuclei.

In the s-process neutron capture chain, the time between successive neutron captures is sufficiently long for the product nucleus to beta decay to a stable system. Within the r-process, the time between neutron captures is too short to permit decays except for very rapid beta transitions. Therefore, the r-process must occur in an environment that has a high density of neutrons. The s-process typically occurs in red giant stars. The r-process occurs in a variety of astronomical events, including supernovae explosions and stellar mergers.

Binary neutron star or neutron star and stellar-mass black hole mergers can form a massive rotating torus around a spinning black hole[1]. The matter ejected from these structures and from supernovae explosions is an important source of rapid neutron capture (r-process) nucleosynthesis[1]. Fully understanding the r-process requires knowledge of the properties of neutron excess nuclei involved in creating heavy nuclear systems. Unfortunately, the majority of these neutron excess systems have never been studied[2].

Closing this knowledge gap was a motivation for funding facilities for rare-isotope beams (FRIB) constructed at research facilities located around the world. These facilities are located at RIKEN (Japan)[3,4], GSI (Germany)[5], and Michigan State University (US)[6]. The FRIB facilities enable a new class of experiments to determine the physical properties needed by theoretical models to determine the structure of unstable neutron excess nuclei. Theoretical studies would complement the forthcoming experiments that will provide critical information on the unstable nuclei that must be understood in order to explain nuclear abundances observed in the universe[2]. In particular, the study of neutron excess systems and their decay properties are significant considerations in understanding the r-process, and its importance in producing the observed elements in the universe.

The study of neutron excess systems is also important for studying nuclear decay properties, nuclear structure under extreme conditions, and nuclear reaction mechanisms. Existing theoretical models have not been extensively applied to many of these neutron excess nuclei.

This paper attempts to partially fill the void by calculating the decay properties of neutron excess systems that are important in nucleosynthesis. These theoretical studies should also assist in planning future experiments associated with neutron excess systems that are far removed from the line of stability.

Neutron excess nuclei that merit study occur throughout the periodic table[2-7] including nuclei in the $Z \leq 32$ range[7]. Although neutron excess nuclei occur throughout the periodic table, this paper focuses on phosphorous systems as part of a continuing investigation of neutron excess nuclei that are of potential astrophysical significance[8-16]. Previous publications addressed neutron excess calcium[8], iron[9], fluorine[10], zinc[11], neon[12], sodium[13], magnesium[14], aluminum[15], and silicon[16] systems.

The study of light nuclear systems, including phosphorous, is important for a comprehensive astrophysical interpretation of nucleosynthesis. For example, Terasawa et al.[17] studied the role of light neutron-rich nuclei during r-process nucleosynthesis in supernovae. Specifically, Ref. 17 noted that light neutron excess systems can significantly affect the heavy-element abundances.

Recent studies emphasize the importance of studying phosphorous isotopes as well as their astrophysical significance[18-24]. These studies include both theoretical as well as experimental efforts.

Goosman and Alburger[18] investigated the ^{35}P system. ^{35}P was produced via the $^{18}O(^{19}F, 2p)$ reaction induced by 42-46-MeV ^{19}F ions, and the $^{36}S(t, \alpha)^{35}P$ reaction via 3.4-MeV tritons. These results are consistent with the expected mass and spin-parity ($1/2^+$) of the ^{35}P ground state. The predicted half-life for ^{35}P is 60±4 sec, where the error arises from the uncertainty in the measured mass. This calculated half-life is close to the measured value of 48±1.4 sec. These calculations should be very useful to experimentalists searching for previously unknown neutron excess nuclei.

The neutron-rich nuclei ^{33}Si, ^{34}Si, ^{35}Si, ^{35}P, ^{36}P, ^{37}S and ^{38}S were investigated by Mayer et al.[19]. Ref. 19 investigated the ^{14}C and ^{18}O induced transfer reactions on ^{36}S using a magnetic spectrograph and a position sensitive focal plane gas detector. The previously unknown mass of the isotope ^{35}Si was determined. The accuracy of the mass excess values of the isotopes ^{33}Si, ^{34}Si, ^{35}P and ^{36}P have been improved. Excited states of the isotopes ^{33}Si, ^{34}Si, ^{35}P, ^{36}P and ^{38}S were identified.

Ref. 20 investigated yrast states in ^{34}P using the $^{18}O(^{18}O, pn)$ reaction at energies of 20, 24, 25, 30, and 44 MeV. The ^{34}P level scheme was investigated utilizing the Doppler-shift attenuation method by detecting decay protons in coincidence with one or more γ rays. The results provide a clearer picture of the evolution of nuclear structure with increasing neutron number.

Nowacki1 and Poves[21] investigated the ^{35}P system. The neutron-rich isotopes with Z ≤ 20 (including ^{35}P), in particular those with neutron numbers around N = 28, have been the focus of a considerable experimental and theoretical effort. Shell-model calculations using effective interactions were utilized to determine many of the properties of these nuclei. This includes the disappearance of the N = 28 shell closure for Z ≤ 16 including phosphorous systems with excess neutrons.

Bender[22] investigated the nuclear structure of neutron rich ^{34}P using in-beam gamma ray spectroscopy. Ref. 22 investigates the increasing dominance of negative parity intruder states in the neutron-rich nuclei in the sd shell as N approaches 20. These systems suggest evidence of an evolving shell structure. By systematically examining the structure of isotopes with increasing N filling the neutron $1d_{3/2}$ level, the results highlight a number of intruder configurations. These intruder configurations come from the decrease in energy to promote a neutron across the N = 20 shell gap. Such studies are needed to help in the refinement of the shell model and increase its predictive power for nuclei far from stability.

Ref. 23 performed calculations of energy level properties of the phosphorous isotopic chain with A = 31 (N=15) to 35 (N=20) determined with shell model calculations. A comparison was made between the calculated results and the available experimental data. The calculated energy spectrum is in good agreement with the available experimental data.

Ref. 24 used the shell model to investigate the complete energy spectra of both positive and negative parity states of ^{33}S, ^{30}P, and $^{33\text{-}35}P$. The results were in reasonable agreement with experiment demonstrating the applicability of theoretical models to describe the energy levels of phosphorous nuclear systems.

Refs. 18-24 have both theoretical nuclear physics as well as astrophysical importance in predicting the production of neutron excess phosphorous nuclei. The continuing interest in neutron excess systems suggests the importance of evaluating phosphorous systems considerably heavier that those investigated in Refs. 18 – 24. In particular, this paper investigates ^{26}P – ^{54}P that span a much greater range than investigated in previous calculations.

2.0 Calculational Methodology

A variety of models could be applied to the investigation of neutron excess nuclei. These vary in sophistication, but the proposed model utilizes a basic single-particle approach. This is a reasonable first step because there are uncertainties in the nuclear potential that likely are more significant than the limitations introduced by a single-particle approach.

Since the method for calculating single-particle energies in a spherically symmetric potential is well-established only salient features are provided. The model used to describe the particle plus core system represents an application of the standard method of Lukasiak and Sobiczewski[25] and Petrovich et. al.[26].

The binding energy E_{NLSJ} of a particle in the field of a nuclear core is obtained by solving the radial Schrödinger Equation

$$\left[\frac{\hbar^2}{2\mu}\left(\frac{d^2}{dr^2} - \frac{L(L+1)}{r^2}\right) - E_{NLSJ} - V_{LSJ}(r)\right]U_{NLSJ}(r) = 0 \quad (1)$$

where r is the radial coordinate defining the relative motion of the nuclear core and the particle; $V_{LSJ}(r)$ is the model interaction; E_{NLSJ} is the core plus particle binding energy; $U_{NLSJ}(r)$ is the radial wave function; and L, S, and J are the orbital, spin, and total angular momentum quantum numbers, respectively. N is the radial quantum number, and μ is the reduced mass.

The method of searching for E_{NLSJ} is provided by Brown, Gunn, and Gould[27], and the methodology of Ref. 28 is utilized to obtain a converged solution. Refs. 8 – 16 and 26 provide a more complete description of the model, its numerical solution, and further definition of the individual terms appearing in Eq. 1.

3.0 Nuclear Interaction

Nuclear stability with respect to alpha decay, beta decay, positron decay, and electron capture is addressed using the method previously published by the author and coworkers[8-16, 26] that is similar to the approach of Ref. 29. The single-particle level spectrum is generated using a Woods-Saxon potential. Parameters of the potential are obtained from a fit to the single-particle energy levels in ^{209}Pb and ^{209}Bi performed by Rost[30]. The central potential strength of the Rost interaction[30] has a standard form and can be explicitly defined as

$$V_0 = 51.6\left[1 \pm 0.73\frac{N-Z}{A}\right] \quad (2)$$

where the upper (lower) sign applies to protons (neutrons). The remaining parameters were held constant and are given by Rost[30]: r_0 = 1.262 (1.295) fm, r_{so} = 0.908 (1.194) fm, a = 0.70 (0.70) fm, and γ = 17.5 (28.2) for protons (neutrons)[26,30]. The spin-orbit interaction strength V_{so} is related to γ by the relationship[30]:

$$V_{so} = \frac{\gamma V_0}{180} \quad (3)$$

The scaling relationships of Eqs. 2 and 3 yield reasonable fits to observed single-particles levels in ^{120}Sn and ^{138}Ba. The pairing correction term of Blomqvist and Wahlborn[31] is used in the calculations presented herein. The pairing correction improves the predicted energies of occupied levels in ^{120}Sn, ^{138}Ba, and ^{208}Pb[26].

When applied to specific nuclei, this methodology requires modification. For example, Ray and Hodgson[32] note that

^{40}Ca and ^{48}Ca require different potentials to properly fit their single-particle level structure. Schwierz, Wiedenhöver, and Volya[33] also investigated ^{40}Ca and ^{48}Ca and noted that a proper fit to the single-particle levels required a different potential for each energy level. Difficulties in the selection of an appropriate potential is an additional motivation for the utilization of a single-particle model, and was noted in studies of neutron excess calcium[8], iron[9], fluorine[10], zinc[11], neon[12], sodium[13], magnesium[14], aluminum[15], and silicon[16] nuclei. Similar issues also apply to phosphorous systems.

In view of the results of Refs. 32 and 33, the following modification is made to obtain the phosphorous potential strength (V_A):

$$V_A = 51.6\lambda\left[1 \pm 0.73\frac{N-Z}{A}\right][1 \pm a(A)]MeV \quad (4)$$

where λ is a potential strength multiplier that is selected to ensure consistency with available data, and a(A) is a constant that is introduced to account for the variations in potential strength with A[32,33]. In previous neutron excess nuclei calculations for calcium[8], iron[9], and zinc[11], a value of λ = 1.0 was utilized. A λ value of 1.5 for fluorine[10], neon[12], sodium[13], magnesium[14], aluminum[15] and silicon[16] was determined by the available experimental data[34-36]. Given the proximity to the A = 12 - 15 systems, a value of λ = 1.5 is also utilized for phosphorous. Since the paper's primary purpose is investigation of the neutron excess nuclei, determining a common a(A) value for the heaviest phosphorous systems is desirable.

The heaviest mass A = 16 isotope[34-36] suggested experimentally is ^{44}P. Given the expected order of energy levels, ^{44}P would have a $2p_{3/2}$ neutron single-particle level structure. Isotopes heavier than ^{44}P would require filling of the $2p_{3/2}$, $2p_{1/2}$, and $1f_{5/2}$ neutron single-particle levels. The possibility of bound phosphorous isotopes with A ≥ 44 is addressed in subsequent discussion. Calculations incorporated into the Japanese nuclear data compilation[36] provide calculated half-lives for ^{45}P, ^{46}P, ^{47}P, and ^{49}P.

4.0 Calculation of Half-Lives

Using Eq. 4, single-particle levels are calculated for A ≥ 26 phosphorous isotopes. A ≥ 26 phosphorous nuclei were evaluated for stability with respect to alpha decay, beta decay, positron decay, and electron capture. These calculations were performed to ensure that the nuclear structure contained no interloping states or structural defects, and that any decay modes in conflict with data were identified.

The decay modes and half-lives of 54 ≥ A ≥ 26 phosphorous isotopes are summarized in Table 1, and compared to available data[34-36] and calculations incorporated in the Japanese data compilation[36]. The alpha decay energies are calculated using the relationship based on Ref. 37.

$$Q_\alpha = 28.3MeV - 2S_n - 2S_p \quad (5)$$

where S_n and S_p are the binding energies of the last occupied neutron and proton single-particle levels, respectively.

Alpha decay half-lives can be estimated from Q_α using standard relationships[25]. Fortunately, no alpha decay modes occurred in the Table 1 summary of 54 ≥ A ≥ 26 phosphorous isotope decay properties.

The beta decay half-lives are determined following the log ft methodology of Wong[37]. Allowed (first forbidden) transition half-lives were derived using the values of log ft = 5 (8). Given the uncertainties in the calculated level energies, second and higher order forbidden transitions were not determined. Positron and electron capture half-lives were determined following the approach of Ref. 25.

5.0 Model Issues

Spherical single-particle energy level calculations produce reasonable results for alpha, beta, positron, and electron capture transitions[8-16, 29-33]. Neutron excess phosphorous isotopes have the potential to decay via neutron emission modes. However, these decays have not been observed in phosphorous [34-36]. The single-particle model is not the best approach for neutron emission calculations, and these decay modes are not included in this paper. Therefore, the results for the heaviest neutron excess phosphorous nuclei only include the alpha decay, beta decay, positron decay, and electron capture modes. Except as noted previously, the single-particle model should provide reasonable results for the systems considered in the paper.

6.0 Results and Discussion

Using Eq. 4, the a(A) value was varied in increments of 0.0001 to assess the applicability of the proposed model to predict the decay properties of most 54 ≥ A ≥ 26 phosphorous isotopes. If a system half-life deviated from the trends in neighboring nuclei, an increment of 0.00001 was utilized. In view of uncertainties in the model and associated interaction, a smaller increment was not deemed to be justified for most phosphorous systems.

The issues associated with fitting all calcium, iron, fluorine, zinc, neon, sodium, magnesium, aluminum, and silicon nuclei with a single potential[32-33] were noted in Refs. 8-16. These considerations are also applicable to the phosphorous systems considered in this paper.

Table 1 summarizes the complete set of 54 ≥ A ≥ 26 phosphorous isotopes considered in this paper. The lighter 54 ≥ A ≥ 26 phosphorous isotopes fill the $1d_{5/2}$ (^{26}P – ^{29} P), $2s_{1/2}$ (^{30}P – ^{31}P), $1d_{3/2}$ (^{32}P – ^{35}P), $1f_{7/2}$ (^{36}P – ^{43}P), and $2p_{3/2}$ (^{44}P) neutron single-particle levels. These systems are the heaviest phosphorous systems noted in Ref. 34 - 36 that have been observed experimentally. Given the extrapolation used in formulating the single-particle potential of Eq. 4, the results become more uncertain due to the paucity of data for A>44 phosphorous isotopes. The heavier 54 ≥ A ≥ 26 phosphorous isotopes that fill the $2p_{3/2}$, $2p_{1/2}$, and $1f_{5/2}$ neutron single-particle levels, and are also summarized in Table 1. These systems represent the heaviest possible neutron excess systems that would occur in the Z=15 system.

Table 1 Calculated Single-Particle and Experimental Decay Properties of Phosphorus with 26 ≤ A ≤ 54

Nuclide	a(A)	Half-Life (Decay Mode)	
		Experiment[a,b]	This Work
^{26}P	-0.0283	44 ms (β^+)[a]	44.0 ms (β^+)[c]
^{27}P	-0.0249	0.3 s (β^+)[a]	0.300 s (β^+)[c]
^{28}P	-0.0094	0.270 s (β^+)[a]	0.270 s (β^+)[c]
^{29}P	+0.0214	4.14 s (β^+)[a]	4.15 s (β^+)[d]
^{30}P	+0.0354	2.5 min (β^+)[a]	2.50 min (β^+)[d]
^{31}P	+0.0300	Stable[a]	Stable
^{32}P	-0.01282	14.28 d(β^-)[a]	14.29 d(β^-)[e]
^{33}P	-0.00762	25.3 d (β^-)[a]	25.1 d (β^-)[e]
^{34}P	+0.0148	12.4 s (β^-)[a]	12.4 s (β^-)[e]
^{35}P	-0.0159	47 s (β^-)[a]	47.1 s (β^-)[e]
^{36}P	-0.0116	5.7 s (β^-)[a]	5.69 s (β^-)[f]
^{37}P	-0.0154	2.3 s (β^-)[a]	2.30 s (β^-)[f]
^{38}P	-0.0052	0.6 s (β^-)[a]	0.599 s (β^-)[f]
^{39}P	-0.0013	0.28 s (β^-)[a]	0.280 s (β^-)[f]
^{40}P	+0.0027	0.15 s (β^-)[a]	0.150 s (β^-)[f]

Table 1(continues) Calculated Single-Particle and Experimental Decay Properties of Phosphorus with 26 ≤ A ≤ 54

Nuclide	a(A)	Half-Life (Decay Mode)	
		Experiment[a,b]	This Work
^{41}P	-0.0013	0.11 s (β^-)[a]	0.110 s (β^-)[f]
^{42}P	+0.0178	48 ms (β^-)[a]	48.0 ms (β^-)[f]
^{43}P	+0.0174	36 ms (β^-)[a]	36.0 ms (β^-)[f]
^{44}P	+0.0400	18 ms (β^-)[a]	18.0 ms (β^-)[f]
^{45}P	+0.0626	[g,h]	10.1 ms (β^-)[f]
^{46}P	+0.0852	[g,i]	6.08 ms (β^-)[f]
^{47}P	+0.1078	[g,j]	3.90 ms (β^-)[f]
^{48}P	+0.1304	[g]	2.62 ms (β^-)[f]
^{49}P	+0.1530	[g,k]	1.83 ms (β^-)[f]
^{50}P	+0.1756	[g]	1.06 ms (β^-)[l]
^{51}P	+0.1982	[g]	0.812 ms (β^-)[l]
^{52}P	+0.2208	[g]	0.633 ms (β^-)[l]
^{53}P	+0.2434	[g]	0.502 ms (β^-)[l]
^{54}P	+0.2660	[g]	0.404 ms (β^-)[l]

Table 1(continues) Calculated Single-Particle and Experimental Decay Properties of Phosphorus with 26 ≤ A ≤ 54

[a]Ref. 34.

[b]Ref. 35.

[c]Allowed $1d_{5/2}$(p) to $1d_{5/2}$(n) positron decay transition.

[d]Allowed $2s_{1/2}$(p) to $2s_{1/2}$(n) positron decay transition.

[e]Allowed $1d_{3/2}$(n) to $1d_{3/2}$(p) beta decay transition.

[f]Allowed $2s_{1/2}$(n) to $2s_{1/2}$(p) beta decay transition.

[g] No data provided in Ref. 34 - 36.

[h] The Japanese data compilation[36] notes a calculated value of 6.105 ms for ^{45}P.

[i] The Japanese data compilation[36] notes a calculated value of 4.72 ms for ^{46}P.

[j] The Japanese data compilation[36] notes a calculated value of 3.05 ms for ^{47}P.

[k] The Japanese data compilation[36] notes a calculated value of 1.52 ms for ^{49}P.

[l]Allowed $1f_{5/2}$(n) to $1f_{7/2}$(p) beta decay transition.

The neutron excess systems summarized in Table 1 were based on an evaluation of alpha, beta, electron capture, and positron decay modes. Spontaneous fission half-lives were also evaluated, but are considerably larger that the aforementioned decay modes.

Other decay modes that could possibly occur in neutron excess systems (e.g., n and 2n) are not readily evaluated using a single particle model, and were not evaluated. The results of Table 1 must be viewed with this limitation. However, since the neutron decay modes tend to be much shorter than the alpha, beta, electron capture, and positron decay modes[34-36], the model results provide upper bounds on the half-lives of neutron excess phosphorous isotopes.

6.1 44 ≥ A ≥ 26 Phosphorous Isotopes with Experimental Half-Life Data

The ^{26}P – ^{29}P systems fill the $1d_{5/2}$ neutron shell. ^{26}P – ^{29}P are best fit with a(A) values between -0.0283 and 0.0214 with an average value of about -0.0103. ^{30}P and ^{31}P fill the $2s_{1/2}$ neutron shell, and are best fit with a(A) value of 0.0354 and 0.0300, respectively. For ^{30}P and ^{31}P, the average a(A) value is 0.0327.

^{32}P – ^{35}P systems were best fit with a(A) values between -0.0159 and 0.0148 with an average value of about -0.0054. The ^{32}P – ^{35}P nuclei fill the $1d_{3/2}$ neutron shell.

The neutron excess ^{36}P – ^{43}P phosphorous systems fill the $1f_{7/2}$ neutron shell. The ^{36}P – ^{43}P systems were best fit with a(A) values between -0.0154 and 0.0178, with an average value of about 0.0004.

^{44}P is the heaviest known neutron excess phosphorous system with an associated a(A) value of 0.0400. ^{44}P's valence neutron is part of the $2p_{3/2}$ shell. There is no experimental half-life data for A > 45 phosphorous systems.

The a(A) values for ^{45}P – ^{54}P systems are based on the decreasing lifetime trends of neutron excess silicon and phosphorous systems[34] . Using the ^{43}P and ^{44}P values, a linear extrapolation was utilized to obtain the a(A) values for ^{45}P - ^{54}P. The derived values are listed in Table 1.

Table 1 lists the half-life of the limiting decay transition (i.e., the transition that has the shortest decay half-life). For example, ^{38}P has five beta decay transitions that are possible within the scope of the aforementioned single-particle model (i.e., allowed $1d_{5/2}$(n) to $1d_{3/2}$(p) [7.79 s]), allowed $2s_{1/2}$(n) to $2s_{1/2}$(p) [0.599 s], allowed $1d_{3/2}$(n) to $1d_{3/2}$(p) [0.642 s]), allowed $1f_{7/2}$(n) to $1f_{7/2}$(p) [0.956 s]), and allowed $1f_{7/2}$(n) to $1f_{5/2}$(p) [5.75 min]). For ^{38}P the limiting beta decay mode is the allowed $2s_{1/2}$(n) to $2s_{1/2}$(p) [0.599 s] transition.

As noted in Table 1, the model predicts the proper decay mode for the known 44 ≥ A ≥ 26 phosphorous[34-36] systems. The results for the known nuclei summarized in Table 1 suggest that the model predictions of the neutron excess systems are reasonably credible.

For nuclei filling the $1d_{5/2}$ neutron shell, model predictions for ^{26}P - ^{29}P are within about 0.25% of the experimental half-lives[34]. ^{26}P - ^{28}P decay via positron emission through allowed $1d_{5/2}$(p) to $1d_{5/2}$(n) transitions. The ^{29}P nucleus decays via positron emission through allowed $2s_{1/2}$(p) to $2s_{1/2}$(n) transitions..

The ^{30}P and ^{31}P systems fill the $2s_{1/2}$ neutron shell. Calculations suggest that ^{30}P decays by positron emission through an allowed $2s_{1/2}$(p) to $2s_{1/2}$(n) transition in agreement with the experimental decay mode and half-life. ^{31}P is predicted to be a stable nucleus in agreement with Ref. 34.

^{32}P – ^{35}P fill the $1d_{3/2}$ neutron shell. The ^{32}P – ^{35}P systems decay by an allowed $1d_{3/2}$(n) to $1d_{3/2}$(p) beta decay transition, and their beta decay half-lives are within about 1% of the measured values[34].

The $1f_{7/2}$ systems, ^{36}P - ^{43}P are within 0.2% of their respective experimental beta decay half-lives[34]. These systems decay by an allowed $2s_{1/2}$(n) to $2s_{1/2}$(p) beta decay transition.

The ^{44}P system initiates filling of the $2p_{3/2}$ neutron single particle level. This is the heaviest phosphorous system that has a measured decay half-life values and beta decay transition information[34]. The ^{44}P system decays by an allowed $2s_{1/2}$(n) to $2s_{1/2}$(p) beta decay transition. The calculated half-life and decay mode are in agreement with Ref. 34.

6.2 54 ≥ A ≥ 45 Phosphorous Isotopes without Experimental Half-Life Data

As noted in the previous section, the a(A) values for 54 ≥ A ≥ 45 phosphorous isotopes were derived from a fit based on the half-lives of ^{43}P and ^{44}P. ^{43}P and ^{44}P are the heaviest phosphorous isotopes measured experimentally[34]. This approach is consistent with the a(A) extrapolation methodology noted in Refs. 8 – 16. The a(A) values for 54 ≥ A ≥ 45 phosphorous systems are provided in Table 1.

Table 1 also summarizes calculated single-particle decay properties of phosphorous systems with 54 ≥ A ≥ 45. Although experimental data for 54 ≥ A ≥ 45 phosphorous systems are not available[34-36], these are nuclei of interest in astrophysical applications[1-24].

The existence of 54 ≥ A ≥ 45 phosphorous systems, as predicted by the proposed model, is dependent on the characteristics of the interaction of Eq. 4. Although the existence of some of these systems may be an artifact of the model interaction, their study is of critical importance in understanding the role of neutron excess phosphorous systems in nucleosynthesis.

The ^{45}P – ^{47}P systems complete filling of the $2p_{3/2}$ neutron shell. These systems also decay through an allowed $2s_{1/2}$(n) to $2s_{1/2}$(p) beta decay transition. The ^{45}P – ^{47}P beta decay half-lives decrease from 10.1 to 3.90 ms, respectively. Japanese Data Compilation calculations[36] for ^{45}P – ^{47}P are also consistent with the model results.

The ^{48}P and ^{49}P systems fill the $2p_{1/2}$ neutron shell. These systems decay through an allowed $2s_{1/2}$(n) to $2s_{1/2}$(p) beta decay transition. The ^{48}P and ^{49}P half-lives are 2.62 and 1.83 ms, respectively. Japanese Data Compilation calculation[36] for ^{49}P is also consistent with the model results.

$1f_{5/2}$ is the last bound neutron shell in phosphorous. ^{50}P – ^{54}P systems partially fill the $1f_{5/2}$ neutron shell. These systems decay through an allowed $1f_{5/2}$(n) to $1f_{7/2}$(p) beta decay transition. The ^{50}P – ^{54}P beta decay half-lives decrease from 1.06 to 0.404 ms.

No phosphorous isotopes with A > 54 are predicted by the model. This occurs because the $1f_{5/2}$ neutron single-particle level is the last bound neutron state, and only 39 neutrons are bound in phosphorous systems. However, in view of the model potential uncertainties, the calculated properties of the heaviest phosphorous systems summarized in Table 1 are not definitive.

The predicted A = 45 – 54 phosphorous isotopes have no experimental half-life data, but the model predicts beta decay half-lives in the range of 0.404 – 10.1 ms. Based on calculations in Z = 9 - 14, 20, 26, and 30 systems[8-16] and calculations summarized in the Japanese Data Compilation[36], the results summarized in this paper likely overestimate the beta decay half-lives of these neutron excess phosphorous nuclei. The model results are also likely to be an overestimate of the half-lives because the single-particle level calculations do not evaluate the short-lived neutron decay modes in the A = 45 – 54 phosphorous nuclei.

7.0 Conclusions

Single-particle level calculations suggest that neutron excess silicon isotopes terminate with ^{54}P and filling of the $1f_{5/2}$

neutron single-particle level. The 45 ≤ A ≤ 54 phosphorous systems have predicted beta decay half-lives in the 0.404 – 10.1 ms range, and likely overestimate the actual half-life values.

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