Commentary

Why Is Gravitational Mass Equal to Inertial Mass?

Qing-Ping Ma[1]

1. Department of Finance, Accounting and Economics, University of Nottingham Ningbo China, China

This paper investigates why gravitational mass is equal to inertial mass, which has been considered a cornerstone of modern physics and is somehow mysterious. By examining how the concept of mass arose, we show that gravitational mass is equal to inertial mass because there is only one mass, the inertial mass, which is operationally measurable for a given force. The ratio of acceleration caused by gravity between two objects is proportional only to their inertial mass ratio. Newton's law of gravity cannot make gravitational mass operationally measurable because it does not define an exact relationship between gravity and the mass of an object in the way Newton's second law of motion defines the relationship between the mass of an accelerating object and the force exerted on it. Therefore, gravitational mass is equal to inertial mass, which equals a force divided by a directly measurable variable (acceleration), because the empirically determined gravitational constant is chosen to make them equal.

1. Introduction

The concept of mass was first formally introduced by Newton[1] as a basic concept in mechanics. The mass has two key properties: providing an object's inertia to resist forces exerted on it and interacting with another object's mass to produce gravity. In the late nineteenth century, Thomson[2] found that the electric and magnetic fields produced by charged particles might provide inertia and behave like mass. Heaviside[3], Lorentz[4], Thomson[5], Searle[6], and Abraham[7] further developed the concept of electromagnetic mass due to electromagnetic energy and even considered electromagnetic energy as the possible origin of mass. Thomson[5] noticed that electromagnetic masses of the bodies depend on their speed and when a charged object's velocity v equals the speed of light c, its mass becomes infinite. Searle[6] gave the precise formula of electromagnetic energy of a moving charged object

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$$E_{em}^{v} = E_{em}\left[\frac{1}{\beta}\ln\frac{1+\beta}{1-\beta} - 1\right]. \tag{1}$$

In Eq. (1), E_{em} is the electromagnetic energy at rest, $\beta = \frac{v}{c}$. He also concluded, "when v = c the energy becomes infinite, so that it would seem to be impossible to make a charged body move at a greater speed than that of light."

With more research in this field, Lorentz[8] derived the longitudinal mass of a moving charged body as k^3/ε times that of its rest mass and its transverse as k/ε times that of its rest mass, where $k = 1/\sqrt{1 - v^2/c^2}$ and ε is a coefficient to be determined. Kaufmann[9] and Abraham[10] derived a different longitudinal mass function from Searl's electromagnetic energy formula,

$$m_L = \frac{3}{4}m_{em} \bullet \frac{1}{\beta^2}\left[-\frac{1}{\beta}\ln\left(\frac{1+\beta}{1-\beta}\right) + \frac{2}{1-\beta^2}\right]. \tag{2}$$

Abraham[10] also derived a transverse different from Lorentz's,

$$m_T = \frac{3}{4}m_{em} \bullet \frac{1}{\beta^2}\left[\frac{1+\beta^2}{2\beta}\ln\left(\frac{1+\beta}{1-\beta}\right) - 1\right]. \tag{3}$$

Lorentz[11] showed that ε is unity and obtained transverse and longitudinal masses as

$$m_T = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}, \tag{4}$$

$$m_L = \frac{m_0}{\left(\sqrt{1 - \frac{v^2}{c^2}}\right)^3}. \tag{5}$$

Bucherer[12] and Langevin[13] also derived competing longitudinal and transverse mass formulae by assuming that the electron contracts in the line of motion and expands perpendicular to it, so that the volume remains constant:

$$m_L = \frac{m_{em}\left(1 - \frac{v^2}{3c^2}\right)}{\left(\sqrt{1 - \frac{v^2}{c^2}}\right)^{8/3}}, \tag{6}$$

$$m_T = \frac{m_{em}}{\left(\sqrt{1 - \frac{v^2}{c^2}}\right)^{2/3}}. \tag{7}$$

Experiments confirmed the apparent longitudinal and transverse masses in moving bodies but were not precise enough to differentiate them in the early years[9][14][15][16][17][18][19][20]. By then, physicists had not considered whether objects should have two masses: an inertial mass to resist forces exerted on them and a gravitational mass to interact with other masses to produce gravity.

Einstein[21] introduced the complete physical equivalence of a gravitational field and a corresponding acceleration of the reference system, which is interpreted as the equivalence of gravitational and inertial masses. Now the concept that gravitational and inertial masses are equivalent is a cornerstone in the field of physics and plays a pivotal role in our understanding of the fundamental forces of nature. With the acceptance of special and general relativity by the physics community, early studies by Galilei[22] on free-falling objects and Newton[1] on pendulums composed of different materials are reinterpreted as examining the relationship between gravitational and inertial masses. The experiment to measure the Earth's gravity to various materials by Eötvös[23] is also described as an investigation of the equivalence between gravitational and inertial masses. Popular science writings often present the equivalence between the two masses as mysterious. Some modern researchers think there are three types of mass: inertial mass, passive gravitational mass (weight) reacting on a given gravitational field, and active gravitational mass creating a gravitational field[24].

Although the equivalence of gravitational and inertial masses is widely accepted, Ma[25] argues that gravitational mass equals inertial mass because only inertial mass is measurable for a given force and the gravitational constant is chosen to make the two equal. This paper aims to investigate why gravitational mass equals inertial mass. The rest of the paper is organized as follows. Section two examines the concept of mass in classical physics as defined by Newton's second law of motion; Section three investigates whether Newton's law of gravity can uniquely determine a gravitational mass; Section four discusses the present findings and concludes.

2. The concept and definition of mass in classical physics

In classical mechanics, Newton[1] first defined mass as the quantity of matter in his *Mathematical Principles of Natural Philosophy*. Mass plays a central role in Newtonian mechanics; it is a quantitative measure of an object's resistance to acceleration (inertia) and a determinant of gravitational force between two objects. Newton's second law states that an object's acceleration a is equal to the force exerted on it, F, divided by its mass m:

$$a = \frac{F}{m}. \tag{8}$$

Since the most basic measurable quantities in physics are time and length in space, a can be more readily obtained by measuring distance and time intervals. Eq. (8) defines mass in terms of force exerted on it and acceleration; it also defines force in terms of an object's mass and acceleration.

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If we examine how the three variables in Newton's second law of motion are determined, we find that acceleration is exogenously determined while force and mass are endogenously determined. Acceleration is the rate of velocity change primarily determined by length and time measurements, we need to know neither the force involved nor the object's mass to compute its acceleration. While acceleration can be measured independently of mass and force, an object's mass and the force exerted on it cannot be determined without each other. From a logical and philosophical point of view, Newton's second law defines the relationship between mass and force for a given external variable acceleration, so it is true by definition. Being true by definition is a logical truth, that needs no empirical confirmation.

Since force and mass are defined by Newton's second law of motion, they are interdependent so Newton's second law cannot be verified experimentally without additional assumptions about mass and force. The defined relationship between mass and force for a given acceleration is true for mechanical and all other forces including gravitational and electromagnetic forces. An object's mass and acceleration also determine the strength of gravity or electromagnetic interaction it receives. If we call this mass in Newton's second law inertial mass, inertial mass is the only mass that is clearly and uniquely defined.

From a logical point of view, Newton's second law could define a different relationship between mass and force such as

$a = \frac{F}{m^2}$.

Although such a second law of motion would change the function forms of most laws and theorems in physics, physics could still explain the physical world as current physics does. The linear relationship between mass and force in Newton's second law is more a consequence of humans′ preference for linearity because of its convenience than a necessity. A nonlinear relationship between them would change definitions of key concepts such as momentum, *P*, which currently is the product of an object's mass *m* and its velocity ***v***:

$$P = mv. \tag{9}$$

It would also change the definition of an object's kinetic energy, *K*, which in Newtonian mechanics is

$$K = \frac{1}{2}mv^2. \tag{10}$$

Had the mass been determined exogenously or operationally like we measure acceleration without involving force, we would not have the current form of Newton's second law. When we define mass independently of force, we may find that Newton's second law is inconsistent with experimental findings. Then, we can only say acceleration is proportional to the force exerted on an object and inversely to its mass,

$$a = k\frac{F}{m}.$$

In the above equation, k is the coefficient to be determined experimentally. Therefore, the relationship between mass and force endogenously defined by Newton's second law is fundamental. The values of other forces, such as gravity or electromagnetic forces, must be measured according to the second law.

In Newtonian mechanics, mass is conserved and can be measured in different settings. If the seemingly same forces cause different accelerations, it must be that the forces are different. Using this conservation of mass, Ma[26] proposes that longitudinal and transverse masses found in experiments[14][10][16][18][19][20] are epiphenomena of reduced electromagnetic forces when charged objects move in electromagnetic fields rather than real changes in mass.

Since the main role of mass in Newton's second law is to resist force, i.e. inertia, conditions that provide resistance to force may be considered apparent "mass" such as electromagnetic energy of electromagnetic fields produced by charged objects[2][3][4]. Stokes[27] has also shown that the inertia of a body increases when it moves in an incompressible perfect fluid. The modern view is that mass is generated via the Higgs field[28].

3. Mass and gravity

Some may argue that Newton's gravitation law also defines the relationship between a force and mass so we can define mass and force accordingly. This view probably motivates the equivalence principle or the equivalence of gravitational and inertial masses. However, contrary to the generally held view that Newton's gravitation law defines a gravitational mass, the law of universal gravitation cannot establish the relationship between mass and force. There are three problems with the generally held view:

First, the value of gravity is still measured by acceleration determined by Newton's second law using inertial mass in theory.

Second, Newton's gravitational law does not define an equality between force and mass. Instead, it describes a proportional relationship with the coefficient to be determined.

Third, Newton's gravitational law defines a proportional relationship between force and the product of two masses, which makes measuring individual mass with this law operationally impossible.

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Newton's law of gravitation states that the gravitational force between two bodies is

$$F = G\frac{m_1 m_2}{r^2}, \tag{11}$$

where $m_1$and m_2 are the masses of the two bodies, *r* is the distance between them, and *G* is the gravitational constant. Although we can use an object's weight to denote its mass and the weights of objects are linearly proportional to their masses on the Earth's surface, gravity cannot determine an object's mass in general. It is simple to prove this because, for any particular values of gravity and distance, gravity is proportional to the product of two masses and an infinite number of combinations of two masses can produce that product.

Gravity can only determine masses relative to a standard mass for a given local dominant mass such as Earth. This is how balances and scales work. Using Newton's second law, we need only a standard force and the exogenously measured acceleration to determine masses. Using Newton's law of gravity, however, we need the force, the second mass, the empirically determined gravitational constant, and the exogenously measured distance to determine masses. Therefore, Newton's law of gravity cannot uniquely determine a mass like Newton's second law does. For example, if the total mass of two objects is *m* and their distance is *r*, different divisions of the total mass will lead to very different gravity values. Let the masses of the two parts be $m_1$and m_2, the gravity between them depends on the ratio of their masses. Divided equally ($m_1 = \frac{m}{2}$ and $m_2 = m/2$),

$F = G\frac{m_1 m_2}{r^2} = G\frac{m^2}{4r^2}, a_1 = a_2 = G\frac{m}{2r^2}.$

One to three ($m_1 = m/4$ and $m_2 = 3m/4$),

$F = G\frac{m_1 m_2}{r^2} = G\frac{3m^2}{16r^2}, a_1 = G\frac{3m}{4r^2}, a_2 = G\frac{m}{4r^2}.$

One to seven ($m_1 = m/8$ and, $m_2 = 7m/8$),

$F = G\frac{m_1 m_2}{r^2} = G\frac{7m^2}{64r^2}, a_1 = G\frac{7m}{8r^2}, a_2 = G\frac{m}{8r^2}.$

There is no unique relationship between the total mass and the gravitational force, or between a single mass and the gravitational force. Different divisions of the total mass lead to various values of acceleration.

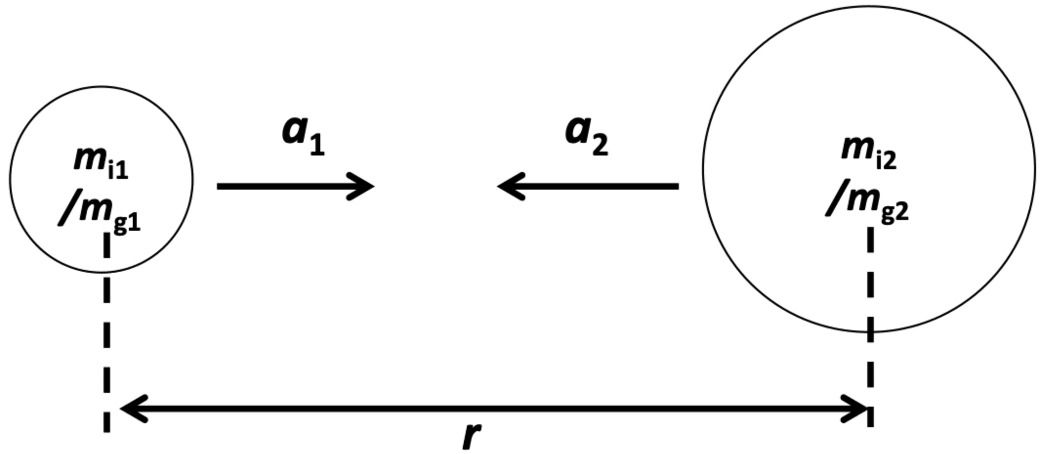

Figure 1. Determination of the relationship between the force and acceleration. Object 1 has mass m_{i1}/m_{g1} and acceleration a_1; Object 2 has mass m_{i2}/m_{g2} and acceleration a_2.

Since an inertial mass and its acceleration uniquely determine a force, gravity between two objects has to be measured by their inertial mass and acceleration. For two objects with gravitational mass m_{g1} and m_{g2} respectively and inertial mass m_{i1} and m_{i2} respectively (Fig.1), their accelerations because of the gravity between them are

$$a_1 = G\frac{m_{g1}m_{g2}}{r^2 m_{i1}}. \tag{12}$$

$$a_2 = G\frac{m_{g1}m_{g2}}{r^2 m_{i2}}. \tag{13}$$

Examining Eqs. (12) and (13), we can see that even if there is gravitational mass and it is different from inertial mass, it is operationally irrelevant because the gravitational constant is empirically determined. If no method can measure gravitational mass exogenously, the empirically determined gravitational constant can always ensure the equality of inertial and gravitational masses and the gravity

$$F_g = G_i\frac{m_{i1}m_{i2}}{r^2} = G_g\frac{m_{g1}m_{g2}}{r^2}. \tag{14}$$

In Eq. (14), G_i is the gravitational constant when inertial mass is used and G_g is the gravitational constant when gravitational mass is used.

Using which mass in the gravity equation does not affect the accelerations caused by the gravity between them. The ratio of acceleration between the two objects is determined by inertial masses only,

$$\frac{a_1}{a_2} = \frac{m_{i2}}{m_{i1}}. \tag{15}$$

Thus, the explicit feature of gravity depends on inertial mass. The ratio between their accelerations is the inverse of the ratio between their inertial masses. Hence the inertial mass rather than gravitational mass determines gravity. Replacing gravitational mass with inertial mass will not affect the outcome of empirical observation so there is no measurable gravitational mass from an operational point of view.

Suppose we have a method to measure gravitational mass and obtain values different from inertial mass. We would obtain a different gravitational constant when we use the gravitational masses to measure the gravitational constant. The new constant will include the difference between the "gravitational mass" and the inertial mass. We may view the equivalence principle as the consequence of empirically determined gravitational constant because we choose a value that makes the two masses equal. If Newton had postulated the gravity law without a constant,

$F_g = \frac{m_{g1} m_{g2}}{r^2}$,

we would have an inequality of gravitational mass m_g and inertial mass m_i, with a relationship

$m_g = \sqrt{G} m_i$.

4. Discussion

Before the advent of Lorentz's ether theory and Einstein's special relativity, what mass is seems clear. According to Newton[1], mass is the quantity of matter in an object. Matter has been a philosophic and physical concept with a very long history. For a long time, people believed that matter could change but would not disappear. The ancient Greek philosopher Empedocles (approx. 490–430 BCE) noted: "For it is impossible for anything to come to be from what is not"[29]. Mikhail Lomonosov proposed the principle of conservation of matter/mass in 1748[30]. Antoine Lavoisier expressed the idea more clearly several years later and demonstrated their validity experimentally[31]. The law of conservation of matter/mass in chemistry and physics states that the mass of an isolated system (closed to all transfers of matter and energy) will remain constant over time. Lorentz's ether theory[11] and Einstein's special relativity[32] no longer think mass is constant.

Aristotle was the first to investigate physics systemically. He thought heavier objects would fall faster and forces kept objects moving. Galilei[22] disproved those Aristotelian views with experiments showing that heavier objects did not fall faster than light ones and forces were not needed to keep objects moving in

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his book published in 1632. Newton[1] formally proposed the law of gravity and observed that the period of a pendulum depended on its mass regardless of material composition. Einstein[21] proposed the equivalence principle. In Einstein's view, gravity is not a force that acts at a distance but rather a manifestation of the curvature of spacetime caused by mass and energy. The mass that causes spacetime to curve (gravitational mass) and the mass that follows the curvature (inertial mass) are the same, so all objects fall at the same rate in a gravitational field. The advent of relativity theory changed people's understanding of mass. Early experiments exploring whether gravity depends on other features of an object are interpreted as examining the equivalence principle. Many experiments have been performed to verify the equivalence principle[33][34][35]. Some researchers question the wide validity of the equivalence principle[36][37].

This paper analyzes how mass is defined and used in Newtonian mechanics and shows that only inertial mass is uniquely and operationally determined. While Newton's second law gives the relationship between mass and force for a given acceleration, the law of gravity cannot compute a mass for a given gravity, gravitational constant, and distance. Therefore, the equivalence between gravitational and inertial masses arises from the inability to measure gravitational mass uniquely. While Newton's second law is more a definition, Newton's gravitation law is an empirical law whose application depends on the empirically determined gravitational constant. Many different empirical laws govern forces that do not involve mass, such as electromagnetic force, but measuring them also needs Newton's second law to interpret measurement results.

The present paper also demonstrates that the empirically determined gravitational constant ensures inertial mass satisfying Newton's law of gravity as long as the gravitational mass is linearly proportional to the inertial mass. If there is a method to measure the gravitational mass independently of the inertial mass, the two masses can differ. However, there is no method to measure gravitational mass independently of the inertial mass, inertial mass can be evaluated without gravitational mass. Therefore, operationally there is only one inertial mass. Conceptually, dividing types of mass into inertial, active gravitational, and passive gravitational[24] may help understand the topic, but in practice only inertial mass can be measured independently without involving other masses.

In conclusion, gravitational mass cannot be uniquely determined and gravity needs inertial mass to measure the acceleration it causes, which makes gravitational mass operationally unmeasurable. The empirical determination of the gravitational constant can always set gravitational mass equal to inertial mass. Operationally there is only one mass, the inertial mass defined by Newton's second law of motion and equal to the force divided by acceleration.

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