

Peer Review

Review of: "Aspects of a Randomly Growing Cluster in $\mathbb{R}^2$, $d \geq 2$ "

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1. Summary of the Paper

This paper presents a novel random growing cluster model in $\mathbb{R}^d$. % with the resulting points and the length of the minimum spanning tree (MST) bounded.

The cluster model starts at the origin, and each subsequent point is a displacement from a previous point (uniformly chosen). The value of displacements is modeled by Gaussian distributions whose variance decreases as the number of points increases.

The authors have rigorously proven the asymptotic behavior of the resulting points.

Specifically, the authors have shown that each resulting point is bounded almost surely in Theorem 1. Moreover, they have proven that the points behave as if they were simulated from a subset of dimension $\min(d, 1/\alpha)$ in terms of the length of the minimal spanning tree in Theorem 2, where α controls the decay rate of variance.

2. Key Advantages

- Novel: This newly proposed growing cluster model is easy to conduct, with appealing asymptotic behaviors, advancing the frontier of simulation methods.
- Rigorous: The mathematical proofs for Theorems 1 and 2 are rigorous, concise, and efficient (although not easy for me to understand).
- Professional: The entire paper is generally composed clearly, logically, and professionally, and well-organized.

3. Major Concerns and Suggestions

- Theorem 1: The proof of Theorem 1 may be better and easier to follow if the authors could provide a simulation example for illustration (if feasible). For example, for a fixed value of α , conduct

a simulation study, showing that $\rho(\mathcal{X})$ is bounded by L when $n=100, 1000$, and 10000 , or even larger.

- Motivation: The authors may add a short paragraph clarifying the motivation of their study, providing some context and potential application.

4. Minor Issues

- The notation $N(0, \sigma)$ is ambiguous; is σ the variance or the standard deviation?
- There may be a typo in the title; $\mathbb{R}^2$ should be $\mathbb{R}^d$.

5. Recommendation

Regarding the novelty and mathematical rigor, I think it is worth publishing.

Declarations

Potential competing interests: No potential competing interests to declare.