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On Einstein-Bohr Debate and Bell's Theorem

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Abstract

According to quantum mechanics in its current form, quantum randomness is an intrinsic property of the physical world. The legitimacy of such interpretation is the essence of the Einstein-Bohr debate. Bell's theorem implies that local realism should be responsible for the experimental invalidation of Bell inequalities. In this paper, it is shown that quantum randomness is due to the unattainability of precise space and time coordinates, and the failure of Bell inequalities is irrelevant to local realism. Furthermore, by reconsidering Bell's theorem in the context of the mathematical setting for quantum physics, namely, Hilbert space, it is shown that, whether a measurement is performed on a system or not, the logical relation between orthogonal vectors corresponding to mutually exclusive properties of the system must be disjunction ("or") rather than conjunction ("and"). Moreover, if the unattainability of precise space and time coordinates is taken into account, and if disjunction is used to serve as the logical relation between the orthogonal vectors, then it is possible to render quantum mechanics complete in the sense considered by Einstein while not essentially modifying the mathematical setting for quantum physics.

Keywords: Quantum randomness, Uncertainty relation, Quantum superposition, Einstein-Bohr debate, EPR experiment, Bell inequalities, Bell experiments, Bell's theorem, Unattainability of precise time and space coordinates

1 Introduction

The quantum-mechanical description of the physical world at the level of microscopic objects, such as photons and electrons, is based on the notion of "quantum superposition". If the state of an individual microscopic object is described by a quantum superposition, how to interpret *quantum randomness* exhibited in the outcomes obtained by measuring such objects is one of the most controversial issues concerning current quantum theory. According to the standard interpretation given by quantum mechanics in its current form, quantum randomness is an intrinsic property of the physical world rather than caused by

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lack of relevant knowledge or due to imperfections of measuring instruments. The legitimacy of this interpretation is the essence of the Einstein-Bohr debate [1, 2, 3].

The standard interpretation of quantum randomness is closely related to the quantum-mechanical description of the physical world. If the quantum-mechanical description is valid, then the standard interpretation of quantum randomness is legitimate. By showing the purported inherent nature of quantum randomness, Heisenberg's uncertainty relation serves to illustrate quantum randomness observed in the outcomes of measuring individual quantum objects. As we all know, in their celebrated paper [1], Einstein, Podolsky, and Rosen (henceforth EPR) questioned the completeness of the quantum-mechanical description. As shown in the famous thought experiment proposed by EPR in their paper (henceforth *EPR experiment*), if the quantum-mechanical description is complete, then Heisenberg's uncertainty relation and the assumed completeness of the quantum-mechanical description imply a contradiction. Underlying the EPR experiment are the assumptions of freedom of choice, locality, and realism. Locality and realism together are also referred to as the assumptions of local realism. Based on the above assumptions, EPR concluded that the quantum-mechanical description is incomplete. Bohr did not consider anything wrong with the assumption of freedom of choice, but he disagreed with the notion of locality underlying the EPR experiment [2].

Bell inequalities are devised to find a better description of the physical world, more complete than the quantum-mechanical description [4]. Bell experiments aim to test Bell inequalities against quantum mechanics [5]. Hailed as "the most profound discovery of science" [6], Bell's theorem claims that, under the same assumptions adopted in the EPR experiment, the predictions given by Bell inequalities differ significantly from the quantum-mechanical predictions. Ironically, when tested by real experiments with all relevant loopholes closed [7, 8, 9, 10, 11], Bell inequalities are found to be wrong because the predictions of Bell inequalities contradict the experimental facts while the probabilistic predictions of quantum mechanics are always correct. According to the standard interpretation of the experimental invalidation of Bell inequalities, either or both of locality and realism must be wrong and should be abandoned. Thus, "in the way which Einstein would have liked least" [12], Bell experiments purportedly resolved the Einstein-Bohr debate [13].

The present paper aims to reveal the origin of quantum randomness and to question Bell's theorem. Quantum randomness is due to the unattainability of precise space and time coordinates. Bell's theorem is questionable because the experimental invalidation of Bell inequalities is irrelevant to the assumptions adopted by EPR, and because Bell experiments imply a fatal logical flaw. By reconsidering Bell's theorem in the context of the mathematical setting for quantum physics, namely, Hilbert space, it is shown that, whether a measurement is performed on a system or not, the logical relation between orthogonal vectors corresponding to mutually exclusive properties of the system must be disjunction ("or") rather than conjunction ("and"). In addition, if the unattainability of precise space and time coordinates is taken into account, and if disjunction is used to serve as the logical relation between orthogonal vectors in Hilbert space for describing the system, then it is possible to render quantum mechanics complete in the sense considered by EPR, and meanwhile, it is not necessary to change the mathematical setting for quantum physics essentially.

In Section 2, the EPR experiment is revisited. In Sections 3 and 4, the cause of quantum randomness and the failure of Bell inequalities are elucidated in detail, respectively. In Section 5, Bell's theorem is reconsidered in connection with the mathematical setting of quantum physics. In Section 6, the main findings reported in this paper are summarized and briefly discussed with respect to the so-called quantum information technologies, such as quantum computation and quantum communication [14].

2 EPR Experiment Revisited

In the literature, Bell experiments are considered as the modern versions of the EPR experiment. However, the former and the latter are essentially different. In the EPR experiment, the *quantum correlation* between two spatially separated systems is introduced to question not only the completeness of the quantum-mechanical description of the physical world but also the legitimacy of the standard interpretation of quantum randomness. Unfortunately, the concept of quantum correlation introduced in the EPR experiment is confused with the notion of "quantum entanglement" in Bell experiments. The confusion is largely due to the notion of quantum superposition that lies at the heart of the quantum-mechanical description.

The concept of quantum correlation is introduced by EPR with an example. The example is a composite system consisting of two correlated and spatially separated particles. By taking this composite system as an example, EPR aimed to show the incompleteness of the quantum-mechanical description: The composite system is quantum-mechanically described by a wave function expressed in the form of a quantum superposition. Because the assumed completeness of the quantum-mechanical description and Heisenberg's uncertainty relation imply a contradiction, EPR concluded that the quantum-mechanical description is not complete [1]. Clearly, the purpose of the EPR experiment is to question the legitimacy of describing the physical world based on the notion of quantum superposition. In sharp contrast to the EPR experiment, Bell experiments take what EPR questioned for granted; this issue will be discussed in Subsection 4.2. See also [15].

Now consider, in detail, the role played by quantum correlation between the two systems, particle I and particle II, in the EPR experiment. The particles had previously interacted, then separated, and no longer interact with each other after separation. By the assumption of freedom of choice, one can choose to measure either of two complementary observables, such as the momentum or position of a particle, say, particle I. Of course, because the momentum and position of the particle are both continuous observables, the measurement can only yield an approximation to the corresponding outcome. Mathematically, for a continuous observable, approximations to the outcomes obtained by measurements may constitute a sequence of more and more accurate results, tending to a unique, definite value, which is the limit of the sequence. However, the limit itself is practically unattainable by measurement. Actually, no precise values of continuous, real-valued observables can be obtained by measurement, although outcomes of measuring such continuous observables are often referred to as approximated values. Strictly speaking, an "approximated value" is a small interval. Nevertheless, when there is no possibility of confusion, for ease of exposition, an approximation to an outcome of measuring a continuous observable may still be called a value.

Thus, from the measured outcome of particle I, the value of the same observable of particle II can be obtained by prediction without measurement because of the correlation between the particles. By the assumption of locality, after the particles are separated far away, anything that happened to one of the particles will not in any way affect the other particle. As a result, measuring particle I will not in any way disturb particle II. Consequently, if the quantum-mechanical description is complete, then definite values may be assigned to both the momentum and position of particle II by the assumption of realism, although what can be obtained by measurements is merely the corresponding approximations. However, according to Heisenberg's uncertainty relation, definite values cannot both be assigned to the momentum and position of the same particle. This is the contradiction implied by Heisenberg's uncertainty relation and the assumed completeness of the quantum-mechanical description of the composite system, as shown by EPR [1]. The revealed contradiction indicates that the quantum-mechanical description of the physical world is incomplete.

The EPR experiment may be simplified without assuming freedom of choice. Of course, freedom of choice is a valid assumption, which is just no longer needed in the simplified EPR experiment. Nevertheless, local realism must still be assumed. By the assumptions of local realism, one may measure the momentum of particle I and the position of particle II. Because the particles are correlated, one may obtain both the value of the position for particle I and the value of the momentum for particle II by prediction from the measured outcomes corresponding to position of particle II and momentum of particle I, respectively. Consequently, each particle can be measured without disturbing in any way the other particle, while definite values of position and momentum may be assigned to both particles. In other words, under the assumptions of local realism, we see again the contradiction implied by the assumed completeness of the quantum-mechanical description of the composite system and Heisenberg's uncertainty relation.

In general, the formal derivation of Heisenberg's uncertainty relation relies on the notion of "commutator". For an individual quantum object, denote by $\hat{A}$ and $\hat{B}$ the operators associated with observables α and β, respectively; their standard deviations are $\Delta\alpha$ and $\Delta\beta$. Let ψ be the state function of the system, where ψ is expressed in the form of a quantum superposition. By definition, the commutator of $\hat{A}$ and $\hat{B}$ is

$$[\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A}.$$

The uncertainty relation then is given by

$$\Delta\alpha\Delta\beta \geq \frac{1}{2} \left| \left\langle [\hat{A}, \hat{B}] \right\rangle \right| .$$

The equality holds if and only if the commutator vanishes identically.

In the early development of quantum theory, the meaning of Heisenberg's uncertainty relation is interpreted as follows: Because of the disturbance caused by simultaneous measurements of the position and momentum of the same particle, the momentum and position of a particle cannot be simultaneously measured to arbitrary precision. Actually, no such disturbance is involved here. As indicated clearly by the standard deviations $\Delta\alpha$, $\Delta\beta$, and the mean value $\left\langle [\hat{A}, \hat{B}] \right\rangle$, the measurements are performed on different but identically prepared particles of the same kind. According to quantum theory in its current form, the only impediment to the simultaneous determination of values for α and β by measurements is the so-called non-commutativity of $\hat{A}$ and $\hat{B}$, namely,

$$[\hat{A}, \hat{B}]\psi \neq 0.$$

However, the non-commutativity has nothing to do with simultaneous determination of values for α and β by measurements, because there exists a physical constraint imposed on measuring individual quantum objects, which is much more stringent than the non-commutativity. According to this constraint, *the same single quantum object can at most be measured only once.* Therefore, after measuring either α or β, but not both, it is impossible to measure the quantum object anymore. As can be readily seen, the commutator $[\hat{A}, \hat{B}]$ violates the constraint imposed on measuring individual quantum objects by allowing the *same single* quantum object to be measured more than once.

Nevertheless, the constraint imposed on measuring individual quantum objects is not necessarily an impediment to assigning definite values to α and β before measurements, as shown correctly by EPR [1]. On the other hand, assigning definite values to α and β before measurements may not necessarily imply that the values of α and β are obtainable by measurements. In addition, Heisenberg's uncertainty relation is expressed in terms of mathematical expectations and standard deviations concerning position and momentum corresponding to measurement results of a large number of *different* particles of the same kind. Such statistics are useful for us to understand a *typical* particle representative of a given kind of individual particles, but may not be appropriate to describe a *particular* particle. Therefore, using Heisenberg's uncertainty relation for describing the random behavior of a *particular* particle may make little sense. It may also be worth noting that, in the EPR experiment, the same particle will not be measured more than once under the assumption of freedom of choice.

Therefore, the quantum-mechanical description of the physical world appears to be indeed incomplete as shown by EPR; however, the incompleteness of the quantum-mechanical description of the physical world and whether the quantum-mechanical predictions are correct are two different issues. There is no experimental evidence for the quantum-mechanical predictions failing to be correct. On the contrary, the quantum-mechanical predictions are always in agreement with the experimental facts.

3 Origin of Quantum Randomness

Many experiments have already confirmed the correctness of the probabilistic predictions given by quantum mechanics. Nevertheless, the standard interpretation of quantum randomness is still debatable. According to the standard interpretation, quantum randomness is an intrinsic property of the physical world, and hence quantum mechanics is inherently probabilistic. The standard interpretation of quantum randomness is closely related to the quantum-mechanical description of the physical world. The legitimacy of the former is implied by the validity of the latter. Equivalently, if the standard interpretation of quantum randomness is illegitimate, then the quantum-mechanical description of the physical world is invalid. However, showing the incompleteness of the quantum-mechanical description may not necessarily negate the standard interpretation of quantum randomness. To resolve the Einstein-Bohr debate reasonably, it is necessary to negate the standard interpretation of quantum randomness. Revealing the origin of quantum randomness can negate the standard interpretation.

Physical quantities all exist in *space and time of the real world.* To measure physical quantities, we must model space and time of the real world mathematically. The hint to reveal the origin of quantum randomness then can only be found in such mathematical models. The mathematical model of space in which we live and measure physical quantities is the three-dimensional Euclidean space $\mathbb{R}^3$ endowed with a metric d. The metric is the usual distance function. The distance between two arbitrary points $\mathbf{r} = (r_1, r_2, r_3)$ and $\mathbf{r}' = (r'_1, r'_2, r'_3)$ in space is given by

$$d(\mathbf{r}, \mathbf{r}') = \sqrt{(r_1 - r'_1)^2 + (r_2 - r'_2)^2 + (r_3 - r'_3)^2}.$$

By definition, $d(\mathbf{r}, \mathbf{r}') = 0$ if and only if $\mathbf{r} = \mathbf{r}'$. The mathematical model of time elapsed in the real world is the set of positive real numbers $\mathbb{R}_+$ equipped with a metric. This metric is the distance function d restricted to $\mathbb{R}_+$. The length between two arbitrary instants $s, t \in \mathbb{R}_+$ then is simply given by the absolute value of their difference $|t - s|$. Induced by the corresponding metrics, $\mathbb{R}^3$ and $\mathbb{R}_+$ are topological spaces with the metric topologies formed by open subsets of $\mathbb{R}^3$ and $\mathbb{R}_+$, respectively. As can be readily seen below, erroneously interpreting quantum randomness as an intrinsic property of the physical world stems from omitting a well-established mathematical fact, i.e., *precise space and time coordinates are unattainable by measurements.* This fact is irrelevant to anything about instruments used for measurements in practice. Although it is always possible to improve the measurement results, the unattainability of precise space and time coordinates will remain forever, and we may have to live with it.

A neighborhood of $\mathbf{r}$ is a set $V(\mathbf{r}) \subset \mathbb{R}^3$ with $\mathbf{r} \in B(\mathbf{r}, \gamma) \subset V(\mathbf{r})$, where $B(\mathbf{r}, \gamma)$ is an open ball with center $\mathbf{r}$ and radius $\gamma > 0$ defined by

$$B(\mathbf{r}, \gamma) = \{\mathbf{r}' \in \mathbb{R}^3 : d(\mathbf{r}, \mathbf{r}') < \gamma\}.$$

A necessary condition to measure the coordinates of $\mathbf{r}$ perfectly precisely is that $\mathbf{r}$ is an isolated point of $\mathbb{R}^3$, i.e., there is a neighborhood $V(\mathbf{r})$ such that

$$\mathbb{R}^3 \cap V(\mathbf{r}) = \{\mathbf{r}\}.$$

However, this condition is false for every $\mathbf{r}$, because for each sufficiently small real number $\gamma > 0$,

$$V(\mathbf{r}) \cap B(\mathbf{r}, \gamma) = B(\mathbf{r}, \gamma) \neq \{\mathbf{r}\}.$$

Similarly, a neighborhood of an arbitrary instant $t \in \mathbb{R}_+$ is an interval $K(t)$ such that t is in an open subset of $K(t)$. The subset is an "open ball" with center t and radius $\gamma < t$. This "open ball" is simply an open interval $(t - \gamma, t + \gamma)$. A necessary condition to obtain the precise coordinate of t by measurement is that t is an isolated point of $\mathbb{R}_+$, i.e., there is an interval $K(t)$ such that

$$\mathbb{R}_+ \cap K(t) = \{t\}.$$

The above condition does not hold for any $t \in \mathbb{R}_+$. In other words, $\mathbb{R}_+$ has no isolated points. To see this, consider an arbitrary $t \in \mathbb{R}_+$. For each sufficiently small $\gamma > 0$,

$$K(t) \cap (t - \gamma, t + \gamma) = (t - \gamma, t + \gamma) \neq \{t\}.$$

Therefore, in no sense can precise space and time coordinates be obtained by measurements. This important fact is the key to understanding quantum randomness. Omitting this fact and misguided by precise but practically unattainable coordinates, one might be allured to characterize a given kind of identically prepared individual quantum objects by using a single quantum object, even though one is aware of the physical constraint imposed on measuring individual quantum objects elucidated in the last section, i.e., the same single quantum object can at most be measured only once. Implying the standard interpretation of quantum randomness, such characterization cannot provide any reasonable explanation about the cause of the randomness exhibited in real experiments with quantum objects.

For example, the randomness exhibited in the behavior of individual photons is due to the unattainability of precise space coordinates, and omitting the unattainability of precise space coordinates will result in using a single photon to characterize identically prepared individual photons, leading to the standard but unreasonable interpretation of quantum randomness. To be specific, consider an experiment with a plane polarized beam of identically prepared individual photons in a sequence $(\nu_k)_{k \geq 1}$. Each of these photons propagates purportedly in the same direction and encounters a polarizer astride its direction of propagation. All the photons in the sequence purportedly have the same polarization direction $\mathbf{r}_0$. The orientation $\mathbf{r}'_a$ of the polarizer is neither parallel nor perpendicular to $\mathbf{r}_0$. It is not difficult to specify an arbitrary direction (or orientation) in space by the precise coordinates of a unique point on a unit sphere $D \subset \mathbb{R}^3$.

$$D = \{\mathbf{r} : d(\mathbf{r}, 0) = 1\}. \tag{1}$$

However, because precise coordinates of each point in $\mathbb{R}^3$ are unattainable by measurements, the *actual* polarization directions $\mathbf{r}_k$ of the k-th photon and the *actual* orientations $\mathbf{r}'_k$ for measuring $\nu_k, k = 1, 2, \cdots$ are all unknown when the experiment is actually performed. In fact, $\mathbf{r}_0 \neq \mathbf{r}_k$ except for at most a finite number of k; otherwise $\mathbf{r}_0$ would equal $\mathbf{r}_k$ for infinitely many k, which implies that the precise coordinates of $\mathbf{r}_0$ can be attained by measurements in actually performed experiments. By no means will this happen in the real world! Therefore, we can only use a small volume $V(\mathbf{r}_0)$ as an approximation, which contains $\mathbf{r}_0$ and $\mathbf{r}_k, k = 1, 2, \cdots$. The volume may be considered as an infinitesimal quantity but *must not be treated as zero.* Similarly, the precise coordinates of $\mathbf{r}'_a$ and $\mathbf{r}'_k$ cannot be obtained by measurements either and $\mathbf{r}'_a \neq \mathbf{r}'_k$ except for at most a finite number of k. As a result, a small volume $V(\mathbf{r}'_a)$ containing $\mathbf{r}'_a$ and $\mathbf{r}'_k, k = 1, 2, \cdots$ has to be used as an approximation.

Consequently, for each k, the precise value of the angle between $\mathbf{r}'_k$ and $\mathbf{r}_k$, denoted by θ_k, is also unknown. Let θ represent a precise value specified for detecting all the photons in $(\nu_k)_{k\geq 1}$, namely, $\theta_k = \theta$ for each k. However, $\theta_k = \theta$ can only hold at most for a finite number of k, because $\mathbf{r}_0 \neq \mathbf{r}_k$ except for at most a finite number of k. In addition, as a continuous quantity, θ is also practically unattainable just like precise space and time coordinates, and a small interval $J(\theta)$ containing θ is what can be obtained by measurement and serves as an approximation to the desired value θ in a real experiment. The precise but unknown values $\theta_k, k = 1, 2, \cdots$ are also contained in $J(\theta)$.

Omitting the unattainability of precise space coordinates implies that the desired value θ is attainable by measurements for detecting each photon in $(\nu_k)_{k\geq 1}$. In a scenario like this, a single photon characterizes all other photons in $(\nu_k)_{k\geq 1}$, making the behavior of the photons inexplicable: There seems to be no way to distinguish ν_i and ν_j in $(\nu_k)_{k\geq 1}$ if $i \neq j$, and yet, each photon behaves randomly rather than deterministically, say, with a fifty-fifty chance to be detected for $\theta = \pi/4$. Where does such randomness come from?

This question can be answered by taking into account the unattainability of precise space coordinates. Because precise space coordinates are all practically unattainable, $\theta_k = \pi/4$ only holds at most for a finite number of k. The desired value $\theta = \pi/4$ and the precise but unknown values θ_k for detecting $\nu_k, k = 1, 2, \cdots$ are all contained in a small interval $J(\pi/4)$. This interval has a strictly positive length, which may be considered as an infinitesimal quantity but *must not be treated as zero.* Therefore, the identically prepared photons behave randomly rather than deterministically.

To analyze such random behavior further, write $\gamma_k = \theta - \theta_k$. For an arbitrarily fixed k in different repetitions of the experiment with $(\nu_k)_{k\geq 1}$, both θ_k and γ_k are random variables with unknown distributions. Actually, it is unnecessary and even impossible to know their distributions. Both $(\theta_k)_{k\geq 1}$ and

$(\gamma_k)_{k\geq 1}$ consist of independent and identically distributed (i.i.d.) random variables because the photons are statistically independent and identically prepared. Denote by γ_0 a random variable identically distributed as γ_k. Each repetition of the experiment then produces a sample path of $(\nu_k)_{k\geq 1}$. Let the sample path be associated with events $\{\gamma_k = 0\}$ and $\{\gamma_k \neq 0\}, k = 1, 2, \cdots$. By the strong law of large numbers, it is not difficult to see $\gamma_0 \neq 0$ almost surely: because $\gamma_k = 0$ only holds at most for a finite number of k at every sample path, and hence the probability of $\{\gamma_0 = 0\}$ must be zero almost surely (in a trial sense). Write

$$X_k = \begin{cases} 1, & \gamma_k = 0 \\ 0, & \gamma_k \neq 0 \end{cases}$$

and

$$Y_k = \begin{cases} 1, & \gamma_k \neq 0 \\ 0, & \gamma_k = 0. \end{cases}$$

Because $(\gamma_k)_{k\geq 1}$ consists of i.i.d. random variables, both $(X_k)_{k\geq 1}$ and $(Y_k)_{k\geq 1}$ are sequences of i.i.d. random variables. Therefore, by applying the strong law of large numbers,

$$\lim_{\ell\to\infty} \frac{\sum_{k=1}^{\ell} X_k}{\ell} = \mathbb{P}(\gamma_0 = 0) = 0,$$

and hence

$$\lim_{\ell\to\infty} \frac{\sum_{k=1}^{\ell} Y_k}{\ell} = \mathbb{P}(\gamma_0 \neq 0) = 1.$$

It is worth noting that the above analysis is entirely based on the axiomatic probability theory formulated by Kolmogorov, which is sufficient to analyze quantum randomness. It is not necessary to use the so-called quantum probability here. In addition, because each photon can at most be detected only once, using any given photon to characterize any other photon implies that precise but practically unattainable space coordinates are taken for granted. Consequently, except the unreasonable, standard interpretation of quantum randomness, it is impossible to infer anything from a detected photon about the random behavior of any other photon. By taking into account the unattainability of precise space coordinates, the randomness exhibited in the Stern-Gerlach experiment with identically prepared spin-1/2 particles can be analyzed in exactly the same way. The analysis can also be generalized to elucidate the randomness observed in optical experiments for testing Bell inequalities, as shown in the next section.

The randomness caused by the unattainability of precise time coordinates can also be analyzed similarly. Some quantum systems are not subject to the constraint imposed on measuring individual quantum objects and can be measured repeatedly. For a system of this kind, omitting the unattainability of precise time coordinates can lead to incorrect explanation about the measurement outcomes observed by experiments, because almost surely different instants of time are mistaken for the same instant, and hence the outcomes obtained by measuring the system at almost surely different instants in different repetitions of the experiment in question are erroneously explained as the outcomes measured at the same instant, making the random behavior of the system inexplicable.

4 Failure of Bell Inequalities

Before the inception of quantum mechanics, scientists considered locality and realism as two fundamental hypotheses for scientific research. However, quantum mechanics seems to be inconsistent with one or both of the hypotheses. The inconsistency appears to be a demonstration that the quantum-mechanical description of the physical world is incomplete [1]. Bell inequalities, as an effort to "reinterpret quantum mechanics in terms of a statistical account of an underlying hidden-variables theory" [16], attempted to describe the physical world in a way consistent with local realism and hence represented a hope of providing a description more complete than the quantum-mechanical description. Unfortunately, this effort was not successful. When tested by experiments against quantum mechanics, the predictions of Bell inequalities not only conflicted with the quantum-mechanical predictions but also contradicted the experimental facts. Ironically, nowadays the majority of the scientific community considers the experimental invalidation of Bell inequalities as a reason for giving up either locality or realism or both, because it is widely believed that the assumptions of local realism should be responsible for the failure of Bell inequalities. However, Bell inequalities actually have nothing to do with local realism. Taking the optical experiment for testing the CHSH inequality as an example, we can readily see that the failure of Bell inequalities is mainly due to the following two factors.

(a) Based on improper counter-factual reasoning, the derivations of Bell inequalities violate the physical constraint imposed on measuring individual quantum objects.

(b) The quantum-mechanical description of the physical world and the standard interpretation of quantum randomness are taken for granted in Bell experiments, resulting in a fatal logical flaw that cannot be explained away as a loophole to be closed.

In the following, Subsection 4.1 and Subsection 4.2 present the detailed explanations of how the above two factors lead to the failure of Bell inequalities.

4.1 Counter-Factual Reasoning and Bell Inequalities

The description of the physical world given by Bell inequalities is not the quantum-mechanical description. The main difference lies in the interpretation of quantum randomness. According to the quantum-mechanical description, quantum randomness is considered an intrinsic property of the physical world. According to the description given by Bell inequalities, a hidden variable is considered as the cause of quantum randomness. Because Bell inequalities are found to be wrong when tested by experiments against quantum mechanics, the interpretation of quantum randomness given by Bell inequalities must also be wrong. However, the failure of Bell inequalities does not necessarily imply that the quantum-mechanical interpretation is correct, because the origin of quantum randomness is the unattainability of precise space and time coordinates, as shown in Section 3. Irrelevant to the assumptions of local realism, the experimental invalidation of Bell inequalities is partly due to violating the physical constraint imposed on measuring individual quantum objects. The violation takes place merely in Bell inequalities because the constraint will not be violated by actually performed experiments. As can be readily seen, the derivations of Bell inequalities are based on correlation functions constructed by mixing actual and counter-factual outcomes. The correlation functions so constructed cannot describe any physical phenomenon in the real world and hence are not physically meaningful. Therefore, Bell inequalities are all physically meaningless.

Consider, for example, the CHSH inequality tested by the optical Bell experiment [5, 7]. In the derivation of this inequality, different pairs (ν_1, ν_2) of correlated photons are purportedly characterized by different values of the hidden variable introduced to interpret the randomness exhibited in the outcomes obtained by measuring different pairs of photons at two spatially separated polarizers [7]. Denote by λ and F the value and distribution of the hidden variable. The outcomes of measuring ν_1 and ν_2 are given by functions A and B corresponding to polarizers I and II, respectively. The functions take either $+1$ or -1 as their values. The outcomes are purportedly determined by λ and the orientations of the polarizers. Each of the polarizers has two different, arbitrarily chosen orientations. Denote by $\mathbf{a}$, $\mathbf{a}'$ and $\mathbf{b}$, $\mathbf{b}'$ the orientations of polarizers I and II, respectively. Hence

$$A(\lambda, \mathbf{a}) = \pm 1,\ A(\lambda, \mathbf{a}') = \pm 1,\ B(\lambda, \mathbf{b}) = \pm 1, \text{ and } B(\lambda, \mathbf{b}') = \pm 1.$$

Let S be a quantity defined as follows.

$$S(\lambda, \mathbf{a}, \mathbf{a}', \mathbf{b}, \mathbf{b}') =$$
$$A(\lambda, \mathbf{a})B(\lambda, \mathbf{b}) - A(\lambda, \mathbf{a})B(\lambda, \mathbf{b}') + A(\lambda, \mathbf{a}')B(\lambda, \mathbf{b}) + A(\lambda, \mathbf{a}')B(\lambda, \mathbf{b}').$$

After a simple inspection, we see

$$S(\lambda, \mathbf{a}, \mathbf{a}', \mathbf{b}, \mathbf{b}') =$$
$$A(\lambda, \mathbf{a})[B(\lambda, \mathbf{b}) - B(\lambda, \mathbf{b}')] + A(\lambda, \mathbf{a}')[B(\lambda, \mathbf{b}) + B(\lambda, \mathbf{b}')] = \pm 2.$$

Integrating S over the set Λ of all values of the hidden variable.

$$\int_{\lambda \in \Lambda} dF(\lambda) S(\lambda, \mathbf{a}, \mathbf{a}', \mathbf{b}, \mathbf{b}') = \pm 2.$$

Suppose the polarizers are in orientations, say, $\mathbf{a}$ and $\mathbf{b}$. The corresponding correlation function of A and B is

$$\mathbb{E}(\mathbf{a}, \mathbf{b}) = \int_{\lambda \in \Lambda} dF(\lambda) A(\lambda, \mathbf{a}) B(\lambda, \mathbf{b}).$$

Consequently, the CHSH inequality is

$$-2 \leq \mathbb{E}(\mathbf{a}, \mathbf{b}) - \mathbb{E}(\mathbf{a}, \mathbf{b}') + \mathbb{E}(\mathbf{a}', \mathbf{b}) + \mathbb{E}(\mathbf{a}', \mathbf{b}') \leq 2.$$

The CHSH inequality allows each component of every pair (ν_1, ν_2) to be measured in two different orientations, which amounts to allowing the same photon to be detected more than once. Thus, the constraint imposed on measuring individual quantum objects is violated. To see this in detail, we can explicitly label S with the given pair (ν_1, ν_2), and label A and B with the corresponding components ν_1 and ν_2. Thus, the outcomes corresponding to different components of (ν_1, ν_2) can be written as $A_{\nu_1}(\lambda, \mathbf{a})$, $A_{\nu_1}(\lambda, \mathbf{a}')$, $B_{\nu_2}(\lambda, \mathbf{b})$, and $B_{\nu_2}(\lambda, \mathbf{b}')$. Accordingly, S takes the following form.

$$S_{(\nu_1,\nu_2)}(\lambda, \mathbf{a}, \mathbf{a}', \mathbf{b}, \mathbf{b}') =$$
$$A_{\nu_1}(\lambda, \mathbf{a})[B_{\nu_2}(\lambda, \mathbf{b}) - B_{\nu_2}(\lambda, \mathbf{b}')] + A_{\nu_1}(\lambda, \mathbf{a}')[B_{\nu_2}(\lambda, \mathbf{b}) + B_{\nu_2}(\lambda, \mathbf{b}')].$$

As indicated clearly by the above expression of S, each component of every pair (ν_1, ν_2) can be detected twice in two different orientations: ν_1 is measured along $\mathbf{a}$ and $\mathbf{a}'$, and ν_2 is measured along $\mathbf{b}$ and $\mathbf{b}'$. However, after ν_1 and ν_2 are measured in whatever directions, they will not be available for detection anymore. For instance, if ν_1 and ν_2 have been detected when polarizer I is in orientation $\mathbf{a}$ and polarizer II is in orientation $\mathbf{b}$, then it is no longer possible to measure either ν_1 or ν_2, and talking about the so-called counter-factual outcomes obtained by measuring ν_1 and ν_2 in any other directions is meaningless, because the same single quantum object cannot be detected more than once, not only in actually performed measurements but also in counter-factual measurements.

Based on improper counter-factual reasoning, actual and counter-factual outcomes are used for constructing the correlation functions not only in the CHSH inequality but also in all other Bell inequalities. Such counter-factual reasoning is improper because it treats the identically prepared pairs of correlated quantum objects of the same kind as the same pair and allows their components to be measured more than once. The counter-factual outcomes involved in the correlation functions purportedly represent objective reality that exists whether or not the corresponding measurements are actually performed. Independently of observation or measurement, objective reality exists indeed; however, mixing actual and counter-factual outcomes makes the correlation functions so constructed physically meaningless because there are no physical phenomena in the real world corresponding to such correlation functions, and hence the correlation functions cannot describe any statistical correlation in the real world. Consequently, Bell inequalities in general and the CHSH inequality in particular make little sense for the description of the physical world. Of course, actually performed measurements will never violate the constraint; the violation of such a constraint takes place only in mathematical descriptions or explanations of the measurement outcomes.

By taking precise but practically unattainable space coordinates for granted, the derivations of Bell inequalities are all based on the correlation functions constructed by mixing actual and counter-factual outcomes based on improper counter-factual reasoning and allow the same single quantum object to be measured more than once. In contrast, under the assumption of freedom of choice, the argument given by EPR neither involves anything constructed by mixing actual and counter-factual outcomes nor implies that the same particle can be measured repeatedly [1]. This may explain why Bell inequalities have nothing to do with the assumption of realism adopted by EPR. As will be elucidated in the following subsection, Bell inequalities are also irrelevant to the assumption of locality adopted by EPR.

4.2 Fatal Logical Flaw in Bell Experiments

Bell experiments aim to test Bell inequalities against quantum mechanics. Bell inequalities are devised to reinterpret quantum mechanics by introducing a hidden variable to account for quantum randomness. As suggested by the meaning of “reinterpret,” reinterpreting quantum mechanics based on a hidden variables theory implies that the basic notions in quantum mechanics, such as quantum superposition and quantum entanglement, remain essentially unchanged. Such notions imply not only the quantum-mechanical description of the physical world but also the standard interpretation of quantum randomness, which are important issues in the Einstein-Bohr debate to be resolved by Bell experiments. However, based on the hidden-variables theory, the reinterpretation of quantum mechanics takes such basic notions for granted. In other words, the validity of the quantum-mechanical description of the physical world and the legitimacy of the standard interpretation of quantum randomness, which are exactly what Einstein argued against in the debate, are presumed in the reinterpretation. As shown above, the reinterpretation actually presumes Bohr’s arguments concerning the corresponding issues in the debate, and hence implies a logical flaw in Bell experiments.

Because of this logical flaw, the fate of Bell inequalities for reinterpreting quantum mechanics is already predetermined by presuming Bohr’s arguments even before Bell experiments are actually performed. The logical flaw is fatal and cannot be explained away as a loophole to be closed. Because Bell inequalities describe the physical world in a way different from quantum mechanics and do not involve any quantum-mechanical notions, the flaw cannot be found in Bell inequalities; it can only be found in the descriptions of identically prepared pairs of correlated quantum objects in Bell experiments for testing Bell inequalities. These descriptions are quantum-mechanical. The logical flaw is also responsible for the failure of Bell inequalities. Because of the logical flaw, the failure of Bell inequalities is believed to be experimental evidence for the existence of physical counterparts of quantum superposition and quantum entanglement. However, as elucidated above, such belief is grounded on the fatal logical flaw and hence questionable.

To be specific, consider the well-known optical Bell experiment for testing the CHSH inequality [7]. By generalizing the analysis presented in Section 3, it can be readily seen that the randomness exhibited in the outcomes of measuring the polarizations of correlated photons actually stems from the unattainability of precise space coordinates. Therefore, the experimental invalidation of Bell inequalities is not experimental evidence for the existence of anything characterized or implied by the purported inherent randomness of the physical world. In particular, quantum superposition and quantum entanglement have no physical counterparts in the real world.

In the optical Bell experiment, a sequence of identically prepared pairs $(\nu_1, \nu_2)_{k\geq 1}$ of correlated photons is generated from the same source, one pair at a time. The photons in each pair counter-propagate purportedly along the same direction in space specified by an arbitrarily chosen z-axis corresponding to a point $\mathbf{r}_z$ in the set D, see Eq.(1). Each pair is quantum-mechanically described by the same entangled state expressed in the form of a quantum superposition [7].

$$|\Psi(\nu_1, \nu_2)\rangle = \frac{1}{\sqrt{2}}[|x, x\rangle + |y, y\rangle].$$

The superposed states are $|x, x\rangle$ and $|y, y\rangle$, where $|x\rangle$ and $|y\rangle$ are linear polarization states. After ν_1 and ν_2 in the same pair are spatially separated, their polarizations are analyzed by linear polarizers I and II in arbitrarily chosen orientations $\mathbf{a}$ and $\mathbf{b}$, perpendicular to the z-axis. Each photon has two distinguishable measurement outcomes, denoted by $+$ and $-$. The outcome depends on whether the linear polarization of the photon is parallel or perpendicular to the orientation of the corresponding polarizer. Let $h_k(\nu_\ell), \ell = 1, 2$ represent the outcome of measuring ν_ℓ in the k-th pair, $k = 1, 2, \cdots$. It is sufficient to focus on a scenario such that the orientations of the two polarizers are parallel, i.e., $\mathbf{a} = \mathbf{b}$. Let $\mathbf{r}'_a \in D$ correspond to the common orientation of the polarizers. In this scenario, a perfect correlation manifests itself between the measurement outcomes of photons in the same pair.

$$h_k(\nu_1) = h_k(\nu_2),\ k = 1, 2, \cdots .$$

The meaning of the above expression is "measurement outcomes on the two sides of '=' are identical." Because each pair is prepared and tested in the same way in a symmetrical configuration,

$$\mathbb{P}[h_k(\nu_1), h_k(\nu_2)] = \mathbb{P}(+, +) = \mathbb{P}(-, -) = \frac{1}{2},$$

which implies

$$\mathbb{P}[h_k(\nu_1) \neq h_k(\nu_2)] = 0.$$

Similar to the analysis in Section 3, denote by Ω the set of measurement outcomes obtained in the optical Bell experiment.

$$\Omega = \{[h_k(\nu_1), h_k(\nu_2)] : k = 1, 2, \cdots\}.$$

Needless to say, the polarizations of photons are measured in space, the real world modeled by the Euclidean space $\mathbb{R}^3$. Because of the unattainability of precise space coordinates, quantum randomness exhibited in Ω is actually due to subjective ignorance of knowledge about precise coordinates of the points in

D representing the *actual* propagating directions $\mathbf{r}_k$ of $(\nu_1, \nu_2)_k$ and the *actual* orientations $\mathbf{r}'_k$ for measuring the k-th pair, $k = 1, 2, \cdots$. Consequently, $\mathbf{r}_k$ and $\mathbf{r}'_k$ are all unknown when the experiment is actually performed. In other words, the measurement outcomes in Ω are random rather than deterministic because they are the polarizations of photons in *different* pairs measured in *almost surely different, unknown orientations* $\mathbf{r}'_k$, and because photons in different pairs counter-propagate along *almost surely different, unknown directions* $\mathbf{r}_k$. The outcomes represent measurement results of *different* pairs because either photon in each pair can at most be measured only once, as required by the constraint imposed on measuring individual quantum objects. Once a photon is registered at a detector, it cannot be detected anymore. The precise coordinates of $\mathbf{r}_k$ and $\mathbf{r}_z$ are all contained in a tiny volume $V(\mathbf{r}_z)$. Furthermore, the outcomes are obtained by measuring the polarizations of photons along *almost surely different, unknown orientations*, because the precise coordinates of the orientations are unattainable. Similarly, the precise coordinates of $\mathbf{r}'_k$ and $\mathbf{r}'_a$ are all contained in a tiny volume $V(\mathbf{r}'_a)$. The volumes $V(\mathbf{r}_z)$ and $V(\mathbf{r}'_a)$ might be treated as infinitesimal quantities but cannot be considered as zero.

The analysis in Section 3 can now be applied straightforwardly to reveal the origin of quantum randomness exhibited in the optical Bell experiment. Revealing the origin of quantum randomness appears to be the only way to show the difference between "quantum entanglement" and quantum correlation. Let θ_k be the precise but unknown value of the angle between $\mathbf{r}'_k$ and $\mathbf{r}_k$. Let θ represent the precise value specified for detecting the photons for all the pairs, namely, $\theta_k = \theta$ for each k. In other words, θ is the desired angle between $\mathbf{r}_z$ and $\mathbf{r}'_a$. For the same reason elucidated in Section 3, $\theta_k = \theta$ can only hold at most for a finite number of k, and θ is practically unattainable. A tiny interval $J(\theta)$ containing θ and precise but unknown values $\theta_k, k = 1, 2, \cdots$ serves as an approximation to the desired value θ in a real experiment. The length of $J(\theta)$ might be considered as an infinitesimal quantity but must not be treated as zero. Based on the above analysis, the same conclusion obtained in Section 3 concerning the origin of quantum randomness can be reached again: the randomness exhibited in the optical Bell experiment is due to the unattainability of precise space coordinates.

According to the quantum-mechanical description given by the entangled state, if no measurement is performed on either photon in the k-th pair for an arbitrary k, then neither ν_1 nor ν_2 in the pair has a definite polarization state. Once a measurement is performed, say, on ν_1, then immediately ν_2 in the same pair attains a definite polarization state identical to the measurement outcome of ν_1. In contrast, according to Einstein's argument, because the two photons in the same pair are spatially separated, measuring one photon will not disturb the other photon in any way. Therefore, Einstein considered such a quantum-mechanical description implying a "spooky action at a distance", and hence contradicting relativity. By interpreting the sudden state change of ν_2 triggered by measuring ν_1 as a result implied by the so-called "non-locality", which is claimed to be a character of quantum mechanics, most physicists believe that Einstein's criticism can be explained away. However, this interpretation cannot tell us why the pairs of correlated photons behave randomly. For example, although the pairs are all identically prepared and tested under the same conditions, if $i \neq j$, the measurement outcomes $[h_i(\nu_1), h_i(\nu_2)]$ may or may not be identical to $[h_j(\nu_1), h_j(\nu_2)]$; however, there seems to be no way to distinguish any one pair from any other pair. According to the standard interpretation, such quantum randomness is an inherent property of the physical world. Actually, by taking the validity of the entangled state for granted, the quantum-mechanical description has already implied the legitimacy of the standard interpretation even before the optical Bell experiment is performed.

Now we can see clearly that Einstein's argument is correct. "Quantum entanglement" is not a meaningful notion. Because the unattainability of precise space coordinates is omitted, quantum randomness is interpreted incorrectly as an intrinsic property of the physical world. As a consequence of this incorrect interpretation, "non-locality" is attached to "quantum entanglement" in Bell experiments to dismiss Einstein's criticism. In fact, "quantum entanglement" in Bell experiments cannot describe any physical phenomenon in the real world. The phenomena purportedly described by "quantum entanglement" in Bell experiments are actually quantum correlations. Unlike the inexplicable "quantum entanglement" represented by the entangled states in Bell experiments, quantum correlation between spatially separated systems is due to physically explainable reasons: The two systems in the EPR experiment are correlated because they were interacted before separation; photons in the same pair in the optical Bell experiment are correlated because they are created by the same source. Different from the misleading notion of "quantum entanglement" in Bell experiments, quantum correlation in the EPR experiment does not need "non-locality" to explain away "spooky action at a distance". So long as the unattainability of precise space coordinates is taken into account, we can interpret quantum randomness observed in the optical Bell experiment reasonably and hence get rid of the inexplicable "quantum entanglement" while avoiding any "spooky action at a distance" disguised as "non-locality."

5 Bell's Theorem and Hilbert Space

Bell's theorem is proved as a consequence of Bell's inequalities. Based on a hidden variables theory, Bell inequalities are derived purportedly under the assumptions of freedom of choice, locality, and realism adopted by EPR. According to Bell's theorem, local theories of natural phenomena formulated within the framework of realism might all be tested by Bell experiments, and the predictions of Bell inequalities must differ significantly from the quantum-mechanical predictions [16]. Were Bell's theorem correct, the experimental invalidation of Bell inequalities would force us to give up either or both of locality and realism. However, as shown in the previous sections, the assumptions adopted by EPR are irrelevant to Bell inequalities, and Bell experiments imply a fatal logical flaw. Consequently, Bell's theorem is problematic and questionable.

In the literature, Bell's theorem is considered a significant advance in understanding the conceptual foundations of quantum mechanics [16]. However, without considering the *mathematical* setting for quantum physics, the *conceptual* foundations of quantum mechanics may not be properly understood. The mathematical setting for quantum physics is Hilbert space. Needless to say, Hilbert space is a very powerful mathematical tool because the concepts involved in its definition are highly abstract. No practical meanings are assigned to the concepts used for defining Hilbert space in general. In a given application, practical meanings may be assigned to the corresponding concepts to define a specific Hilbert space. Consequently, Hilbert space has widespread applications, not only in quantum physics but also in many other fields.

In general, the elements in Hilbert space are abstract vectors. Orthogonality is one of the most important concepts to define Hilbert space. Mathematically, orthogonality is defined by an inner product of two vectors. Because Hilbert space is a natural generalization of Euclidean space, the inner product and the orthogonality defined for abstract vectors in Hilbert space may look similar to the scalar product defined for ordinary Euclidean vectors and the orthogonality defined for orthogonal vectors in ordinary Euclidean space. Except for the similarities to the above notions in Euclidean geometry, abstract vectors and the orthogonality for defining Hilbert space in general are purely mathematical concepts without geometric or any other practical meaning. In particular, there is no need to assign any practical meaning to the orthogonality. Moreover, for Hilbert space in general, it is even unnecessary to specify the logical relation between orthogonal vectors. Of course, for a specific Hilbert space, the logical relation between orthogonal vectors can be conjunction ("and"). In this case, however, orthogonal vectors must not represent mutually exclusive properties of any element in the Hilbert space, as shown below with an example.

The example is the ordinary Euclidean space $\mathbb{R}^3$. With the inner product defined for the Euclidean vectors, $\mathbb{R}^3$ is a Hilbert space. For this Hilbert space, the orthogonal Euclidean vectors do not represent mutually exclusive properties of any geometric object, and the logical relation between the orthogonal vectors is conjunction. However, this does not necessarily imply that, for *any* Hilbert space, the logical relation between orthogonal vectors can *only* be conjunction. The logical relation between orthogonal vectors in a Hilbert space can also be disjunction ("or").

For quantum physics, abstract vectors in a Hilbert space are physical states of a system. Corresponding to alternative outcomes of measurement or observation, orthogonal vectors in the Hilbert space represent orthogonal states of the system. There is nothing wrong with the above meaning assigned to the orthogonality. However, according to the postulates on which quantum mechanics (in its current form) is founded, the system before measurement is *simultaneously* in each orthogonal state, and hence possesses mutually exclusive properties at the *same time*. In other words, using conjunction as the logical relation between the orthogonal vectors is implied by the assumptions in current quantum theory. Stemming from the omission of the unattainability of precise space and time coordinates, the assumptions are problematic. If the meaning assigned to the orthogonality in a Hilbert space for describing a quantum system is that the orthogonal vectors represent *mutually exclusive* properties of the *same system*, the logical relation between the orthogonal vectors must be disjunction, because no system in the real world can possess mutually exclusive properties at the *same time*, whether or not the system is measured or observed. It may be worth emphasizing again that there is nothing wrong with the meaning assigned to the orthogonality for describing a system in quantum physics; the system can of course possess mutually exclusive properties at different times, but the *same system* cannot have mutually exclusive properties at the *same time*. This is why the logical relation between the orthogonal vectors in a Hilbert space that serves to describe a quantum system must be disjunction rather than conjunction.

Described by the notion of quantum superposition, a quantum system purportedly possesses mutually exclusive properties at the same time before measurement; the properties are represented by the corresponding orthogonal vectors in a Hilbert space. Once a measurement is performed on the system, the quantum superposition collapses immediately onto one of the orthogonal states. In other words, beginning initially with conjunction as the logical relation between the orthogonal vectors corresponding to mutually exclusive properties before measurement, the system, as time evolves, ends up inexplicably in one of the orthogonal states after measurement, and the logical relation between orthogonal vectors changes from conjunction to disjunction. This weird change before and after a measurement then raises a question, as John S. Bell put it: How does an "and" get converted into an "or"?

The above question now may be answered as follows: Using conjunction as the logical relation between orthogonal vectors in a Hilbert space for describing a quantum system is due to the problematic assumptions in current quantum theory, and the assumptions result from omitting the unattainability of precise space and time coordinates. As elucidated in the previous sections, the omission of the unattainability of precise space coordinates may result in confusing identically prepared systems of the same kind with the same system. Because of the confusion, the outcomes obtained by measuring different systems of the same kind along almost surely different directions are mistaken for the outcomes obtained by measuring the same system in the same direction. Similarly, because the unattainability of precise time coordinates is omitted, the outcomes measured at almost surely different instants of time in different repetitions of the experiment in question are mistaken for the outcomes measured at the same instant. Quantum randomness in each case above is then erroneously interpreted. By taking into account the unattainability of precise time and space coordinates and using disjunction as the logical relation between orthogonal vectors in a Hilbert space for describing quantum phenomena, quantum randomness can be interpreted reasonably, which is helpful for rendering quantum mechanics complete in the sense considered by EPR; meanwhile, the mathematical setting for quantum physics can remain essentially unchanged.

6 Discussion and Conclusion

In this paper, it is shown that quantum randomness is due to the unattainability of precise space and time coordinates, and Bell's theorem is questionable. If the unattainability of precise space coordinates is omitted, then almost surely different directions along which different systems of the same kind are measured may be mistaken for the same direction along which the same system is measured. Similarly, if the unattainability of precise time coordinates is omitted, then almost surely different instants of time at which a system is measured in different repetitions of the experiment in question may be mistaken for the same instant at which the system is measured. In each case above, the measurement results are erroneously explained, eventually leading to the standard interpretation of quantum randomness. Because the standard interpretation of quantum randomness is illegitimate, the quantum-mechanical description of the physical world is invalid.

Bell's theorem is questionable because the experimental invalidation of Bell inequalities is irrelevant to local realism, and because Bell experiments imply a fatal logical flaw. By reconsidering Bell's theorem in the context of the mathematical setting for quantum physics, namely, Hilbert space, it is shown that, whether a measurement is performed on a system or not, the logical relation between orthogonal vectors corresponding to mutually exclusive properties of the system must be disjunction ("or") rather than conjunction ("and"). No systems possess mutually exclusive properties simultaneously in the real world. Any system purportedly having such properties is due to the omission of the unattainability of precise space and time coordinates. If the unattainability of precise space and time coordinates is taken into account, and if disjunction is used to serve as the logical relation between the orthogonal vectors, then it is possible to render quantum mechanics complete in the sense considered by EPR, while any essential change of the mathematical setting for quantum physics may be unnecessary.

Based on the notion of quantum superposition, the quantum-mechanical description of the physical world implies the standard interpretation of quantum randomness. When Bell asked how an "and" became an "or," he already implicitly assumed the validity of using the notion of quantum superposition to describe the physical world. According to Bell's theorem, local realism should be responsible for the experimental invalidation of Bell inequalities, and hence either or both of locality and realism should be abandoned. Were such conclusions true, they would indeed be philosophically startling. However, the conclusions are wrong and have led to serious consequences in practice. For sciences, renouncing either or both of locality and realism appears to be a disaster; it opened the door to ineligible applications of quantum mechanics represented by quantum information technologies [13], such as quantum computation and quantum communication [14]. Various attempts to realize such unrealizable technologies have already consumed a huge amount of effort, funding, and investment. In such a way that "Einstein would have liked least"[12], Bell experiments purportedly resolved the Einstein-Bohr debate.

The so-called quantum information technologies are all based on the notion of the quantum bit (qubit). A qubit is a quantum superposition, which describes a two-level system with conjunction serving as the logical relation between the superposed orthogonal vectors purportedly representing mutually exclusive properties of the same system at the same time before measurement. It is claimed that the system for realizing the notion of a qubit might be single microscopic objects such as photons [17], or some composite objects [18, 19, 20, 21, 22], or macroscopic objects [23]. The composite systems and macroscopic objects may be measured repeatedly without being destroyed. However, for a system of this kind, because the unattainability of precise time coordinates is omitted, different outcomes obtained by measuring the system at almost surely different instants of time in different repetitions of the experiment in question are mistaken for the outcomes measured at the same instant.

In fact, for any quantum system in the real world, it is not legitimate to use conjunction as the logical relation between the superposed orthogonal vectors in a Hilbert space for describing the system. Whether a measurement is performed on the system or not, the logical relation between the orthogonal vectors must be disjunction, given that such vectors represent mutually exclusive properties of the system. Using conjunction as the logical relation between the orthogonal vectors is due to the problematic assumptions in current quantum theory. Such assumptions stem from the omission of the unattainability of precise space and time coordinates. Therefore, with conjunction serving as the logical relation between the superposed orthogonal vectors, quantum superpositions have no physical counterparts in the real world, and the so-called quantum information technologies are all ineligible applications of quantum mechanics and doomed to failure. See also [24]

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