Research Article

Noninertial Reference Frames in Special Relativity

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Research published in The Physics Teacher by Pepino and Mabile reveals the existence of widespread confusion in academic circles regarding the necessity of general relativity to describe accelerated frames of reference. We analyze the origin of the controversy and clarify the existing confusion based on the conceptual foundations of special and general relativity.

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I. Introduction

In a revealing study,[1] Pepino and Mabile found from a survey of physics faculty and graduate students from more than 22 physics departments of prestigious institutions in the United States and United Kingdom, that only 37% of faculty members and 10% of the graduate students correctly answered *yes* to the question “Is SR [Special Relativity] capable of describing physics in accelerated reference frames? (*yes or no*)”.

It must be clear that the question is not whether special relativity can describe objects in accelerated motion, but whether special relativity can describe spacetime from within accelerated reference frames.

Of course, special relativity, understood as Minkowskian flat spacetime, can be considered a special case of curved spacetime when the Riemann tensor vanishes. So, there is no doubt that general relativity can handle spacetime referred to arbitrary reference frames in flat spacetime.

However, the relevant conceptual question that concerns us here is whether it is “only” through general relativity that we can describe physics in noninertial reference frames when there is no gravitational field.

In section II, we analyze why the answer can be considered controversial. Section III explains the heuristic approach to interpreting general spacetime coordinates. Section IV presents the formal method that permits the precise implementation of the interpretation explained in the previous section according to the local validity of special relativity. Section V discusses an application clarifying some common misunderstandings existing outside specialist circles.

II. The Controversy

As often happens when a subject is controversial, the reason lies in the fact that there exist two possible admissible answers depending on the departing premises.

There are two ways to approach general relativity. One is axiomatically through a rigorous mathematical formulation (differentiable manifolds, tangent bundles, etc.) The other is constructively following a historical path through basic and more intuitive physical principles.

The first case employs a top-down approach, starting from general axioms, where special relativity is contained as a special case. In the second bottom-up approach, general relativity is constructed from special relativity through heuristic concepts such as the principle of equivalence.

The situation is analogous to the case of electromagnetic theory. One can start by postulating Maxwell's equations from the beginning. In this approach, Coulomb's law of electrostatics becomes a mere lemma consequence of the differential Gauss's law $\nabla \cdot \vec{E} = \rho/\epsilon_0$. On the other hand, we can construct Gauss's differential equation using the empirical Coulomb's law and postulate the universal validity of Gauss's differential formulation, transcending purely electrostatic phenomena.

The question of whether general relativity is necessary for describing accelerated reference frames in the absence of gravitation is similar to discussing whether Coulomb's law should be considered a mere mathematical theorem or a fundamental physical law.

When we learn the theory axiomatically through a precise mathematical approach, we gain logical rigor but perhaps at the cost of some heuristic insight.

Both methods have their own advantages and disadvantages, and both views are necessary for a correct appreciation and deep physical understanding.

Making it clear that special relativity can be considered a special case of general relativity, hence, the latter can describe flat spacetime in arbitrary reference frames; here, we explain how special relativity alone suffices to describe noninertial frames of reference through a heuristic approach that employs only basic physical concepts.

III. Relationship between special relativity, noninertial reference frames, and gravitation

Einstein introduced the special theory of relativity in his celebrated 1905 paper.[2] Although special relativity was conceived to deal exclusively with spacetime referred to inertial coordinates through the Lorentz transformation, in the same seminal 1905 article, Einstein laid the key idea for the special relativistic treatment of noninertial frames of reference by predicting the behavior of an accelerated clock.

To avoid misunderstandings, it should be clear that there is no problem with special relativity describing the accelerated motion of the clock as an object. But the question that concerns us here is, how does time behave for an observer attached to an accelerated object? Since special relativity deals only with space and time transformations for observers in inertial motion, the actual time measured by an accelerated clock can not be predicted without an additional assumption. According to Einstein:

> *"It is at once apparent that this result still holds good if the clock moves from A to B in a polygonal line, and also when the points A and B coincide.*
> *If we assume that the result proved for a polygonal line is also valid for a continuously curved line ... "*

Obviously, he was partitioning a continuous, accelerated path into a finite number of polygonal sections, considering each section in inertial motion, although it is not. That enabled Einstein to apply the Lorentz transformation to calculate the instantaneous rate of the clock under acceleration. In other words, Einstein was approaching accelerated motion as mathematicians rectify a curve that is not a straight line.

Note Einstein's explicit acknowledgement of the assumption through his expression "*If we assume...*" From then on, however, he rarely mentioned the assumption explicitly and proceeded to apply special relativity to accelerated reference frames when necessary, although an accelerated system is not inertial, irrespective of whether its coordinate axes may instantaneously coincide with those of a *"Momentarily Comoving Inertial Reference Frame (MCIRF)".*

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Mashhoon appropriately called Einstein's assumption the *"Hypothesis of Locality (HL)."*[3] A particular instance of the HL is the *clock hypothesis,*[4] which is how Einstein implemented it in 1905 to analyze an accelerated clock. [1]

The identification of an accelerated frame with a MCIRF allows the local application of the Lorentz transformation, although an accelerated frame is not inertial. Similarly, the identification of an infinitesimal arc of a curve with a straight segment allows the "local" application of Pythagoras' theorem; $dl^2 = dx^2 + dy^2$, although Δl is not the length of a straight line. This heuristic approach explains why Einstein considered the equivalence principle *"The happiest thought of my life."*[5]

Indeed, although the *"Einstein Equivalence Principle (EEP),"* is usually phrased in terms of "free falling," it is the equivalence between acceleration and gravitation that enabled Einstein to replace a gravitational field with an accelerated reference frame and finally, through the local application of special relativity allowed by the HL, predict some relativistic effects of gravitation, such as the redshift and the bending of light rays, even before finding the complete theory of general relativity.

That the local "infinitesimal" application of special relativity to accelerated frames is at the heart of general relativity is explained in the following authoritative references. One by Einstein himself:[6]

> *The general theory of relativity rests entirely on the premise that each infinitesimal line element of the spacetime manifold physically behaves like the four-dimensional manifold of the special theory of relativity. Thus, there are infinitesimal coordinate systems (inertial systems) with the help of which the ds are to be defined exactly like in the special theory of relativity. The general theory of relativity stands or falls with this interpretation of ds. It depends on the latter just as much as Gauss' infinitesimal geometry of surfaces depends on the premise that an infinitesimal surface element behaves metrically like a flat surface element...*

Einstein did not explicitly mention the intervention of an accelerated frame in his reasoning. The fact that, for Einstein, equating an accelerated reference frame to a MCIRF was *"at once apparent"* led him to pass directly from the reference frame with a gravitational field to a local inertial reference frame without mentioning the intermediate accelerated reference system used to replace gravity (cf. (1), (2), and (3) ahead).

The other authoritative reference, by Misner, Thorne, and Wheeler[7] is more explicit regarding the use of local accelerated reference frames,

> *A tourist in a powered interplanetary rocket feels "gravity." Can a physicist by local effects convince him that his "gravity" is bogus? Never, says Einstein's principle of the local equivalence of gravity and accelerations. But then the physicist will make no errors if he deludes himself into treating true gravity as a local illusion caused by acceleration. Under this delusion, he barges ahead and solves gravitational problems by using special relativity.*

The key point above is *"But then the physicist will make no errors if he deludes himself into treating true gravity as a local illusion caused by acceleration;"* i.e., he treats acceleration with special relativity, and that allows him to describe gravitation through the EEP.

Therefore, it is acceleration that explains gravitation, not the other way around. Thus, from this heuristic viewpoint, using gravitation to describe acceleration leads to circular reasoning.[2]

Unfortunately, the heuristic insight explained by Misner, Thorn, and Wheeler above is often lost when we learn the theory through an exclusive rigorous axiomatic approach. However, that insight is the reason why the experts quoted by Pepino and Mabile assert that general relativity is not necessary to explain the Twin Paradox and that the widespread belief that only general relativity can account for it, because acceleration is involved, is misleading.

Next, we compare the explicit use of the HL when applying the EEP (1) with its implicit application in (2) and (3). Let at a given spacetime point, G stand for the gravitational field, LAF for local accelerated reference frame, MCIRF for momentarily comoving inertial reference frame, and FFF for freely falling reference frame. Then we have the following locally valid equivalences,

$$G \xrightarrow{EEP} LAF \xrightarrow{HL} MCIF \equiv FFF \tag{1}$$

On the other hand, when implicitly assuming the HL, the above chain of implications reduces to,

$$G \xrightarrow{EEP} \quad LAF\,(\text{Misner,Thorne, Wheeler example}) \tag{2}$$
$$G \xrightarrow{EEP} \quad FFF\,(\text{Einstein's elevator example}) \tag{3}$$

In (2), the accelerated frame LAF is approached with special relativity using a MCIRF. In (3), the necessary LAF is not mentioned, and a MCIRF is materialized with a freely falling reference frame. In both cases, the HL is tacitly assumed; otherwise, special relativity, originally conceived to deal only with inertial reference frames, could not be rigorously applied.

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IV. Formal Description of the Local Equivalence Between Inertial Motion and Acceleration

We shall make formally explicit that the local equivalence between inertial motion and acceleration does not require the EEP, and the seemingly widespread belief that relativistic gravitation theory must necessarily be involved in the description of noninertial reference frames is misleading.

The characteristic properties distinguishing the inertial reference frames are the homogeneity and isotropy of space and the homogeneity of time.[8] Further, it can be proved that as a consequence of the physical equivalence of all inertial reference frames, their spacetime coordinates transform linearly,[9], and the differential interval,

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu \quad \text{(repeated indices are summed)} \tag{4}$$

remains invariant.[10] The homogeneity and isotropy properties are also reflected in the fact that $\eta_{\mu\nu} = diag(+1, -1, -1, -1)$ remain constant throughout space and time.

In a noninertial reference frame (or in a gravitational field through the EEP), the characteristic homogeneity and isotropy of spacetime that single out the inertial systems are lost, and spacetime transformations are no longer linear. However, since special relativity is still locally valid, the differential interval ds remains invariant. All of that is formally expressed by a change in the coefficients $\eta_{\mu\nu} \to g_{\mu\nu}$,

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \tag{5}$$

where $g_{\mu\nu}(x^0, x^1, x^2, x^3) \neq const$.

The price we have to pay for this generality is that the spacetime coordinates x^μ lose their global meanings and have a direct intuitive significance only locally through the physical interpretation of the interval ds. To extract this meaning from (5) at a given spacetime point, we must put it in the same form as (4) at that point.

We shall use the convention whereby repeated Greek indices are summed from 0 to 3 and Latin indices from 1 to 3, indicating spatial indices.

Separating the spatial part in the expression of the interval,[11]

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$$
\begin{aligned}
ds^2 = &\quad g_{\mu\nu} dx^\mu dx^\nu \\
= &\quad g_{00}(dx^0)^2 + 2g_{0k}dx^0dx^k + g_{ik}dx^idx^k && (6) \\
= &\quad g_{00}(dx^0)^2 + 2g_{0k}dx^0dx^k + g_{ik}dx^idx^k && (7) \\
= &\quad g_{00}\left((dx^0)^2 + 2\frac{g_{0k}}{g_{00}}dx^0dx^k\right) + g_{ik}dx^idx^k && (8) \\
= &\quad g_{00}\left((dx^0)^2 + 2\frac{g_{0k}}{g_{00}}dx^0dx^k \pm \frac{g_{0i}g_{0k}}{g_{00}^2}dx^idx^k\right) \\
&\quad +g_{ik}dx^idx^k && (9)
\end{aligned}
$$

Taking the minus sign term outside the parentheses, we are left with a perfect square,

$$
\begin{aligned}
ds^2 = &\quad g_{00}\left((dx^0)^2 + 2\frac{g_{0k}}{g_{00}}dx^0dx^k + \frac{g_{0i}g_{0k}}{g_{00}^2}dx^idx^k\right) && (10) \\
&\quad -\frac{g_{0i}g_{0k}}{g_{00}}dx^idx^k + g_{ik}dx^idx^k \\
= &\quad g_{00}\left(dx^0 + \tfrac{g_{0k}}{g_{00}}dx^k\right)^2 - \underbrace{\left(-g_{ik} + \tfrac{g_{0i}g_{0k}}{g_{00}}\right)}_{\gamma_{ik}}dx^idx^k && (11)
\end{aligned}
$$

Expressing (11) in ordinary local inertial coordinates,

$$ds^2 = \quad c^2d\tau^2 - dl^2 \qquad (12)$$

Comparing (11) with (12), the spatial distance is given by,

$$dl^2 = \quad \gamma_{ik}dx^idx^k \qquad (13)$$

and the actual elapsed time measured by physical clocks,

$$d\tau^2 = \quad \frac{g_{00}}{c^2}\left(dx^0 + \tfrac{g_{0k}}{g_{00}}dx^k\right)^2 \qquad (14)$$

The fact that the elapsed time in (14) contains the spatial terms dx^k is because the difference in the clock readings corresponds to different clocks; one at the point x^k and the other at $x^k + dx^k$, and those clocks are not synchronized unless $g_{0k} = 0$ for $k = 1, 2, 3$.[10]

It is also possible to obtain (13) and (14) by exchanging light signals between the spatial positions x^k and $x^k + dx^k$ (cf. Ref.). So, (13) and (14) are not merely mathematical tricks, but acquire direct physical significance through the HL.

When we want the elapsed time in the same spatial point, we must put $dx^k = 0$ in (14),

$$d\tau = \quad \frac{\sqrt{g_{00}}}{c}dx^0 \qquad (15)$$

For a finite time interval marked by a fixed physical clock, we have,

$$\tau_2 - \tau_1 = \quad \int_{x^0_{(1)}}^{x^0_{(2)}} \frac{\sqrt{g_{00}}}{c}dx^0 \qquad (16)$$

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Although it is not a standard denomination, we designate this method of extracting physical distances and actual time rates from the general metric (5) the "Local Metric Formalism (LMF)."

Figure 1 shows the relation among special relativity, the HL, the LMF, and general relativity.

We highlight the fact that to implement the LMF in noninertial reference frames, the EEP and gravitation are not required, and only the local application of special relativity, as allowed by the HL, suffices.

The full structure of general relativity is necessary only to find how the mass-energy distribution determines the $g_{\mu\nu}$; however, we particularly remark that the necessity of (5) and its interpretation are exclusive consequences of special relativity and the HL.

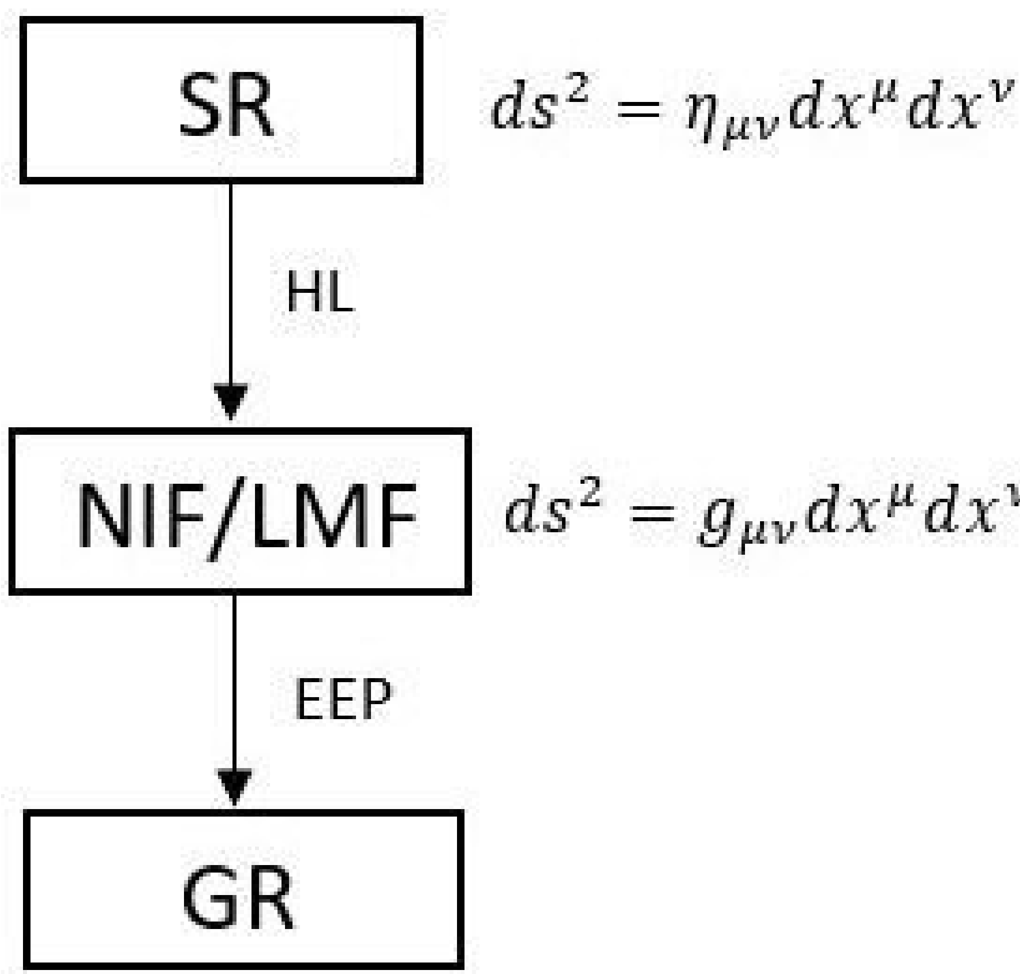

Figure 1. SR ≡ special relativity; HL ≡ hypothesis of locality; NIF ≡ noninertial reference frame; LMF ≡ local metric formalism; EEP ≡ Einstein equivalence principle; GR ≡ general relativity

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V. A Clarifying Example

The incorrect relativistic analysis of rotating noninertial reference frames has led to some erroneous disprovals of special relativity.[3]

One such purported disproof is known as Selleri's paradox,[12][13] and another is the incorrect relativistic analysis of the Sagnac effect.[14][15][16] In both cases, the mistake consists of erroneously assuming global properties of homogeneity and isotropy that are only valid in inertial reference frames.

A concrete flawed argument based on the Sagnac effect claims the following:[14][15]

a. The special relativistic addition of velocities predicts that light speed is invariant for observers fixed along the rim of the rotating platform.
b. Since the clockwise and counterclockwise light beams traverse symmetric paths at the same speeds, an experimenter at C (Fig. 2) rotating with the interferometer must observe both beams arriving simultaneously. Hence, special relativity predicts no displacement of the interference fringes, thus contradicting empirical observation.

One counterargument that advocates of relativity often give for rejecting arguments that involve noninertial reference frames, such as the above, is that in those cases, special relativity is not applicable and only general relativity can provide the correct answer.

Although the formalism of general relativity can indeed account for such cases, our point here is that special relativity alone suffices and that, from a historical and foundational standpoint, that fact was essential to Einstein for the development of general relativity.

A. The Special Relativistic Solution

For the case that concerns us in this section, i.e., the prediction of the Sagnac effect from within the rotating platform, we transform from the coordinates in the inertial reference frame with origin in the disc center (ct', r', ϕ', z') to coordinates fixed in the rotating platform (ct, r, ϕ, z),

$$t = t', \qquad r = r', \qquad \phi = \phi' - \Omega t, \qquad z = z' \tag{17}$$

Here we remark a usual misinterpretation. The time transformation $t = t'$ is sometimes misinterpreted as a Galilean transformation of time, meaning that the variable t inside the platform represents the real time as measured by physical clocks at rest with respect to the rotating platform.[14][15]

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However, that is an incorrect interpretation of the time coordinate. When referred to an arbitrary (noninertial) reference frame, the time coordinate is a mere parameter used to calculate the actual proper time measured by physical clocks in that frame.

Although according to (17), the time coordinate t can be interpreted as the time of the background inertial reference frame, inside the platform it is only an auxiliary parameter that allows the calculation of the true time indicated by physical clocks attached to the rotating platform according to (15) and (16).

For the differential interval ds, (17) gives,

$$ds^2 = \underbrace{\left(1-\beta^2\right)}_{g_{00}} c^2 dt^2 - dr^2 - r^2 d\phi^2 - dz^2 - 2\beta rc\, dt\, d\phi \tag{18}$$

where $\beta = v_0/c$, $v_0 = r\Omega$. The time elapsed on a physical clock fixed in the same position inside the rotating platform is obtained from (16) and (18),

$$\tau_2 - \tau_1 = \int_{x_1^0}^{x_2^0} \frac{\sqrt{g_{00}}}{c}\, dx^0 \tag{19}$$

$$\tau_2 - \tau_1 = \int_{t_1}^{t_2} \frac{dt}{\gamma} \tag{20}$$

To evaluate the Sagnac effect, we need to calculate the elapsed times τ^+ and τ^- in the location of the interferometer.

According to the HL, the light paths must satisfy $ds = 0$, which for $r = R$ leads to,

$$dt^+ = +\frac{Rd\phi^+}{c(1-\beta)}, \qquad d\phi^+ > 0, \text{ (forward beam)} \tag{21}$$

$$dt^- = -\frac{Rd\phi^-}{c(1+\beta)}, \qquad d\phi^- < 0, \text{ (backward beam)} \tag{22}$$

Replacing (21) in (20),

$$\tau^+ = \int_0^{2\pi} \frac{1}{\gamma} \frac{Rd\phi^+}{c(1-\beta)} \tag{23}$$

$$= \frac{1}{\gamma} \frac{L}{c(1-\beta)} \tag{24}$$

Analogously, from (22) and (20),

$$\tau^- = \int_0^{-2\pi} -\frac{1}{\gamma} \frac{Rd\phi^-}{c(1+\beta)} \tag{25}$$

$$= \frac{1}{\gamma} \frac{L}{c(1+\beta)} \tag{26}$$

The displacement of the interference fringes is determined by the difference,

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$$\tau^{+}-\tau^{-}= \qquad \frac{2Lv_0}{c^2\sqrt{1-(\frac{v_0}{c})^2}} \tag{27}$$

Putting $L=2\pi R$ and $v_0=\omega R$, the last result gives the classical first order value of the Sagnac effect

$$\tau^{+}-\tau^{-}= \qquad \frac{4A\omega}{c^2} \tag{28}$$

where $A=\pi R^2$.

That special relativity suffices to explain the Sagnac effect from within the rotating platform is also explained in Ref.[11]

B. Further Clarifications

Although, in Section B, we showed how the formal application of the LMF solves the problem of the Sagnac effect according to a consistent local application of special relativity, point b) of the argument may still seem puzzling.

How is it possible that after following symmetrical paths traversed at the same speeds, the two beams do not arrive simultaneously? As we previously observed, in noninertial reference frames, the global homogeneity and isotropy of space, as well as the homogeneity of time, are no longer valid, and our everyday intuition is not applicable.

The fact that two observers fixed to the platform, such as A and B (Fig. 2), are instantaneously at rest in different inertial frames that are not themselves at rest with respect to each other has unexpected consequences. Also, in the concrete case of a rotating platform, the rotation has a determinate direction that breaks the symmetry along both paths.

Furthermore, the mean speed of the beams calculated with one fixed clock at C does not coincide with the constant speed c measured with local clocks placed along the beam paths. A similar effect also takes place in the gravitational field of the sun. It is known as the Shapiro effect and is considered the fourth classical test of general relativity.[17] This analogy is also mentioned in Ref. [18]

Another usual incorrect interpretation asserts that the local speeds of the beams obtained from (21) and (22) are $c(1-\beta)=c-v$ and $c(1+\beta)=c+v$ respectively. However, that is incorrect because the coordinates times dt^{+} and dt^{-} are not proper physical times (cf. Ref. [19] for a detailed explanation).

An instructive and interesting instance discussing similar issues can be found in an exchange between Robert D. Klauber and T. A. Weber.[20][21]

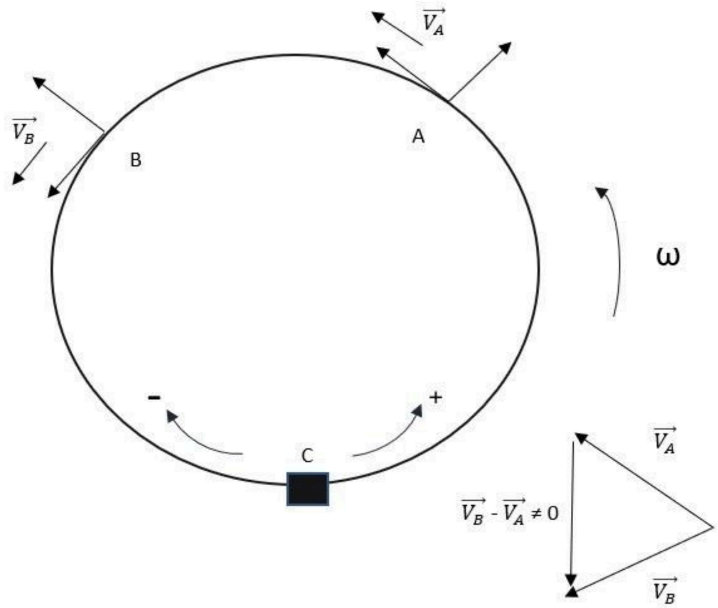

Figure 2. MCIRFs at A and B

VI. Conclusions

Einstein recognized in 1905 that the equivalence between an accelerated reference frame and a momentarily comoving inertial system constitutes an assumption. However, in later applications of the method, Einstein omitted any explicit reference to the assumption and proceeded to apply it as if it were a logical consequence of the theory.

The local application of special relativity to accelerated systems of reference became a self-evident tenet without the need to observe that it is a separate assumption.

Since an accelerated system is not an inertial one, and the special theory of relativity is exclusively formulated for describing space and time within inertial frames of reference, the ensuing confusion among nonspecialists is understandable.

Either accepted as a separate hypothesis or a logical imposition, to avoid the confusion revealed by the Pepino and Mabile study, students should be warned about the local validity of special relativity in noninertial frames of reference at an early stage of exposure to special relativity.

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Footnotes

[1] Mashhoon's HL should not be confused with the "Principle of locality" stating that an object can be influenced only by its immediate surroundings.

[2] We believe this is a deep physical insight that explains why general relativity cannot wholly account for Mach's principle.

[3] In this section, we are assuming a basic familiarity with the Sagnac effec.

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