Research Article

Noninertial Reference Frames in Special Relativity

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Although the special theory of relativity was conceived to address spacetime transformations between inertial reference frames, it is not generally known among beginning students or even among academics not specializing in general relativity that it can be applied to noninertial reference frames. The general perception among non-specialists seems to be that noninertial reference frames can only be described through general relativity. This situation has led to some controversy regarding the necessity of general relativity to address problems involving accelerated systems, such as the twin paradox and the description of rotating reference frames. We analyze the origin of the controversy and propose that to avoid the present confusion, the local validity of special relativity in these general cases should be explicitly and clearly stated at an early stage of the learning process.

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I. Introduction

In a revealing study, [1] Pepino and Mabile found from a survey of physics faculty and graduate students from more than 22 physics departments of prestigious institutions in the United States and United Kingdom, that only 37% of faculty members and 10% of the graduate students correctly answered *yes* to the question "Is SR [Special Relativity] capable of describing physics in accelerated reference frames? (*yes or no*)".

It must be clear that the question is not whether special relativity can describe objects in accelerated motion when observed from an inertial reference frame, but whether special relativity can describe spacetime for observers fixed to an accelerated reference frame. Two classic examples of noninertial reference frames are a rotating platform and a uniformly accelerated observer. The question is, how do

the spacetime coordinates (x^0, x^1, x^2, x^3) of an inertial observer transform to the spacetime coordinates (y^0, y^1, y^2, y^3) of an observer fixed to a noninertial reference frame?

Of course, special relativity, understood as Minkowskian flat spacetime, can be considered a special case of curved spacetime when the Riemann tensor vanishes. So, there is no doubt that general relativity can handle spacetime expressed in terms of noninertial reference frames in flat spacetime.

However, the relevant conceptual question that concerns us here is whether it is "only" through general relativity that we can predict spacetime measurements of noninertial observers when there is no gravitational field.

In section II, we analyze why the answer can be considered controversial. Section III explains the heuristic approach to interpreting general spacetime coordinates. Section IV presents the formal method that permits the precise implementation of the interpretation explained in the previous section according to the general local validity of special relativity. Section V discusses an application clarifying some common misunderstandings existing outside specialist circles.

II. The Controversy

As often happens when a subject is controversial, the reason lies in the fact that there exist two possible admissible answers depending on the departing premises.

There are two ways to approach general relativity. One is axiomatically through a rigorous mathematical formulation based on concepts of differential geometry such as differentiable manifolds, Fiber bundles, etc. The other is constructively following a historical path through basic and more intuitive physical principles.

The first case employs a top-down approach, starting from general axioms, where special relativity is contained as a special case. In the second bottom-up approach, general relativity is "constructed" from special relativity through heuristic concepts such as the principle of equivalence.

It is the second bottom-up perspective that requires the application of special relativity to noninertial frames of reference independently of general relativity. This approach enabled Einstein to predict certain gravitational effects before developing a comprehensive theory of gravitation.

Making it clear that special relativity can be considered a special case of general relativity, hence, the latter can describe flat spacetime in accelarated reference frames; here, we revert to the former axiomatic

approach to explain how special relativity alone suffices to describe noninertial frames of reference through a heuristic principle that employs only basic physical concepts.

III. Relationship between special relativity, noninertial reference frames, and gravitation

Einstein introduced the special theory of relativity in his celebrated 1905 "June" paper.[2] Although special relativity was conceived to deal exclusively with spacetime in inertial coordinates through the Lorentz transformation, in the same seminal 1905 article, Einstein laid the key idea for the special relativistic treatment of noninertial frames of reference by predicting the behavior of an accelerated clock.

To avoid misunderstandings, it should be clear that there is no problem with special relativity describing the accelerated motion of the clock as an object. However, the fundamental question that concerns us here is, can we predict how time passes for an observer attached to an accelerated object?

Since special relativity deals only with inertial observers, to calculare the actual time measured by an accelerated clock, Einstein gives us the following assumption.[2]

> *"It is at once apparent that this result still holds good if the clock moves from A to B in a polygonal line, and also when the points A and B coincide.*
> *If we assume that the result proved for a polygonal line is also valid for a continuously curved line ... "*

He made the clock traverse each polygonal side with constant velocity, i.e., he partitioned a continuous, accelerated path into a finite number of rectilinear sections, considering the clock in inertial motion through each section, although its actual motion is not. That enabled Einstein to apply the Lorentz transformation to calculate the instantaneous rate of the clock under acceleration. In other words, Einstein was approaching accelerated motion as mathematicians rectify a curve that is not a straight line.

Note Einstein's explicit acknowledgment of the assumption through his expression "*If we assume...*" From then on, however, he rarely mentioned the assumption explicitly and proceeded to apply special relativity to accelerated reference frames when necessary, although an accelerated system is not inertial, irrespective of whether its coordinate axes instantaneously coincide with those of a *"Momentarily Comoving Inertial Reference Frame (MCIRF)".*

Mashhoon appropriately called Einstein's assumption the *"Hypothesis of Locality (HL)."* [3] A particular instance of the HL is the *clock hypothesis*,[4][5][6] which is how Einstein implemented it in 1905 to analyze an accelerated clock.[1]

The identification of an accelerated system with a MCIRF allows the local application of the Lorentz transformation. Similarly, the identification of an infinitesimal arc of a curve with a straight segment allows the "local" application of the Pythagorean theorem; $dl^2 = dx^2 + dy^2$, although the arc is not a straight line (Fig.1). This heuristic approach explains why Einstein considered the equivalence principle (EEP) *"The happiest thought of my life."* [7]

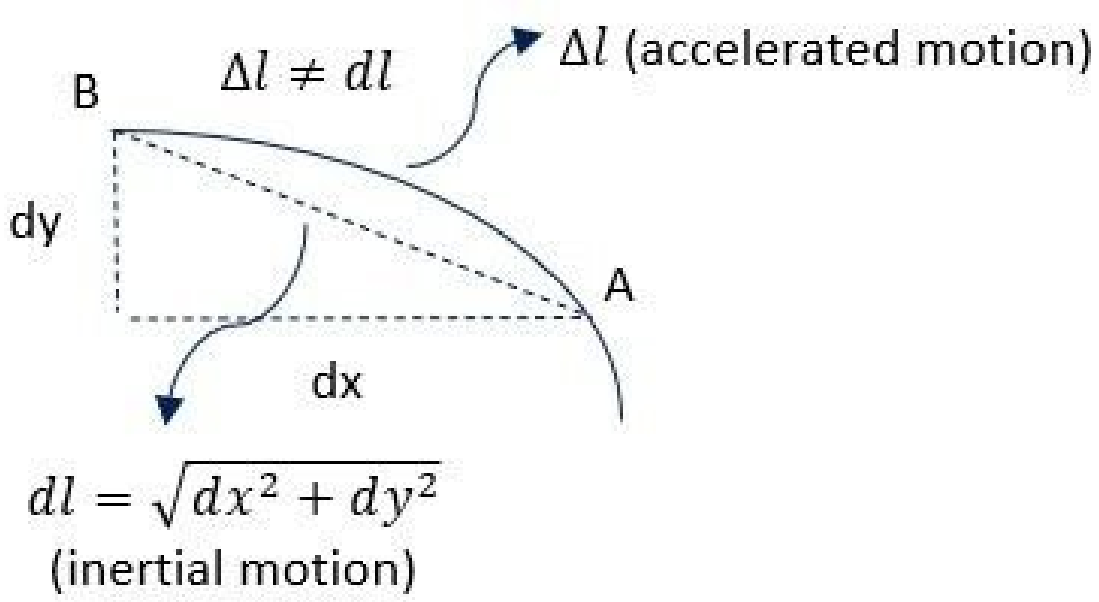


Figure 1. Differential Pythagorean theorem analogy of Einstein's 1905 assumption. In this analogy, the curved segment represents the actual noninertial motion. The rectilinear segment represents the inertial path. While the application of the Pythagorean formula approximates the length of the curve segment, the application of the Lorentz transformation approximates the noninertial motion.

A. Relation between the EEP and the HL

Although the Einstein equivalence principle is usually directly phrased in terms of "free falling", it is the equivalence between acceleration and gravitation that permits the replacement of a gravitational field with an accelerated reference frame, and finally the substitution of the latter with a free-falling frame (FFF). This procedure can be interpreted as the application of the EEP followed by the HL, and it allowed

Einstein to predict some relativistic effects of gravitation, such as the redshift and the bending of light rays, even before finding the complete theory of general relativity.

That the local "infinitesimal" application of special relativity to accelerated frames is at the heart of general relativity is explained in the following authoritative references. One by Einstein himself: [8]

> *The general theory of relativity rests entirely on the premise that each infinitesimal line element of the spacetime manifold physically behaves like the four-dimensional manifold of the special theory of relativity. Thus, there are infinitesimal coordinate systems (inertial systems) with the help of which the ds are to be defined exactly like in the special theory of relativity. The general theory of relativity stands or falls with this interpretation of ds. It depends on the latter just as much as Gauss' infinitesimal geometry of surfaces depends on the premise that an infinitesimal surface element behaves metrically like a flat surface element...*

Although above, Einstein did not mention the motivation for assuming the infinitesimal validity of special relativity, [2] the other authoritative reference, by Misner, Thorne, and Wheeler [9] is more explicit regarding the use of local accelerated reference frames,

> *A tourist in a powered interplanetary rocket feels "gravity." Can a physicist by local effects convince him that his "gravity" is bogus? Never, says Einstein's principle of the local equivalence of gravity and accelerations. But then the physicist will make no errors if he deludes himself into treating true gravity as a local illusion caused by acceleration. Under this delusion, he barges ahead and solves gravitational problems by using special relativity.*

The key point above is *"But then the physicist will make no errors if he deludes himself into treating true gravity as a local illusion caused by acceleration;"* i.e., he replaces gravitation with acceleration and then applies special relativity "locally" in the MCIRF.

Therefore, it is acceleration that explains gravitation, not the other way around. Thus, from this heuristic viewpoint, using gravitation to describe acceleration leads to circular reasoning.

We believe this is a deep physical insight that explains why general relativity cannot wholly account for Mach's principle. This insight remains hidden in the mathematical formulation. Still, the fact that acceleration comes first and gravitation is explained as an outgrowth of acceleration through the equivalence principle is exquisitely explained in the above fable by Misner, Thorne, and Wheeler.

However, the reason we know how to describe physics in an accelerated system is precisely that we apply special relativity to it locally. Thus, as the authors wrote in their textbook, *General relativity is built on special relativity*, and we would add, not the other way around!

That insight is also the reason why the experts quoted by Pepino and Mabile assert that general relativity is not necessary to explain the Twin Paradox and the belief that "only" general relativity can account for it, because acceleration is involved, is misleading.

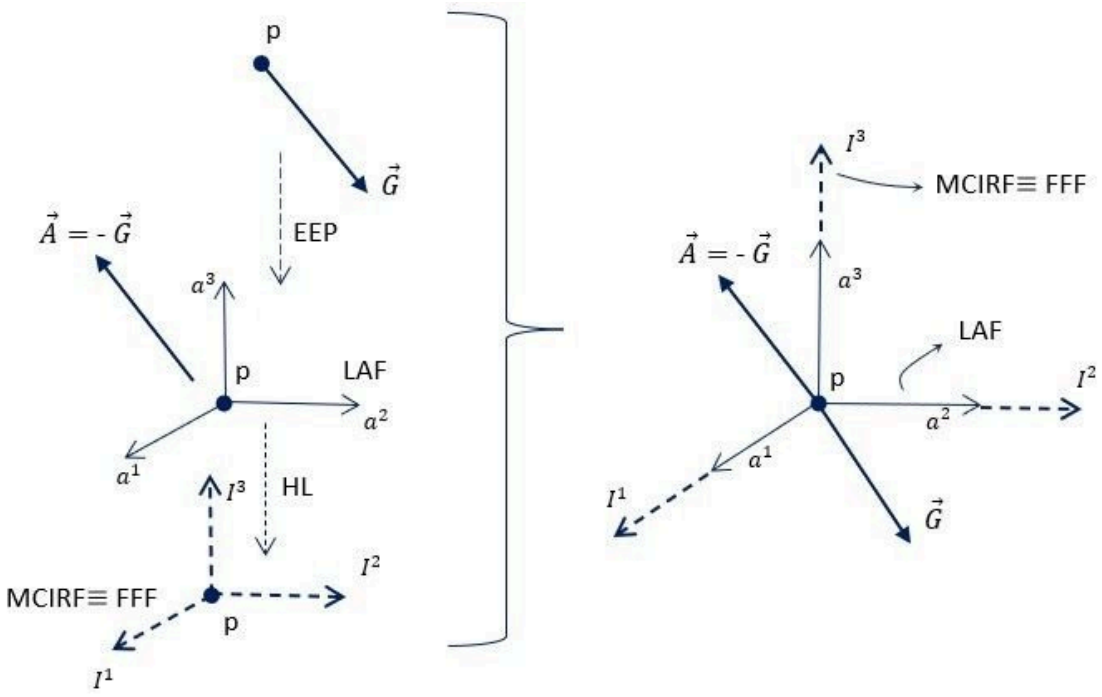


Figure 2. On the left, the elements corresponding to the same spacetime point p are shown separately(the time axis is not shown). In the first step, the field $\vec{G}$ is replaced by an LAF with acceleration $\vec{A}$. In the second step, the LAF is replaced by an $MCIRF$. On the right, it is shown that the LAF and the $MCIRF$ are indeed superposed at p. However, they cannot be considered the same physical system since the LAF is accelerated while the $MCIRF$ is inertial. Although both are rest frames, the fact that one is inertial while the other is not constitutes a subtle difference. [10]

The following arrow-connected schemes show the application of the equivalence principle. In (1), the application of the HL is explicitly shown. LAF stands for "Local Accelerated Frame." Note that the equivalence $MCIF \equiv FFF$ is because the MCIF being inertial does not feel acceleration, therefore is "falling free."

$$G \overset{EEP}{\rightarrow} LAF \overset{HL}{\rightarrow} MCIF \equiv FFF \tag{1}$$

The schemes (2) and (3) represent the usual reasoning where the use of the HL is concealed or implicit.

$$G \overset{EEP}{\rightarrow} \quad LAF\,(\text{Misner, Thorne, Wheeler example}) \tag{2}$$

$$G \overset{EEP}{\rightarrow} \quad FFF\,(\text{Einstein's elevator example}) \tag{3}$$

In (2), the fact that the LAF is described with special relativity is so natural and evident for the authors that the HL is never mentioned. In (3), an inertial free-falling frame is directly used, assuming as obvious that a LAF previously replaces the gravitational field.

IV. Formal Description of the Local Equivalence Between Inertial Motion and Acceleration

We shall make formally explicit that the local equivalence between inertial motion and acceleration does not require the EEP, and the seemingly widespread belief that relativistic gravitation theory must "necessarily" be involved in the description of noninertial reference frames is misleading.

The characteristic properties distinguishing the inertial reference frames are the homogeneity and isotropy of space and the homogeneity of time.[11] Further, it can be proved that as a consequence of the physical equivalence of all inertial reference frames, their spacetime coordinates transform linearly,[12], and the differential interval,

$$ds^2 = \eta_{\alpha\beta} dy^\alpha dy^\beta \quad (\text{repeated indices are summed}) \tag{4}$$

remains invariant.[13] The homogeneity and isotropy properties are also reflected in the fact that $\eta_{\mu\nu} = diag(+1, -1, -1, -1)$ remain constant throughout space and time.

When we admit noninertial reference frames, the characteristic homogeneity and isotropy of space and time that single out the inertial systems are lost, and, as a consequence, the spacetime transformations are no longer linear. It is important to clarify that the loss of homogeneity and isotropy of space and time is an effect or property associated with the system of reference, and not an intrinsic property of the spacetime manifold that remains Minkowskian flat irrespective of the election of the reference frame.

Since special relativity is still locally valid, the differential interval ds remains invariant.

All of that is formally expressed by a change of the coefficients in (4) according to a nonlinear transformation,

$$
\begin{aligned}
y^{\alpha} &= y^{\alpha}(x^0, x^1, x^2, x^3) && (5)\\
dy^{\alpha} &= \frac{\partial y^{\alpha}}{\partial x^{\mu}} dx^{\mu} && (6)\\
ds^2 &= \underbrace{\eta_{\alpha\beta} \frac{\partial y^{\alpha}}{\partial x^{\mu}} \frac{\partial y^{\beta}}{\partial x^{\nu}}}_{g_{\mu\nu}} dx^{\mu} dx^{\nu} && (7)\\
ds^2 &= g_{\mu\nu} dx^{\mu} dx^{\nu} && (8)
\end{aligned}
$$

where $g_{\mu\nu}(x^0, x^1, x^2, x^3) \neq const.$ [3]

The price we have to pay for this generality is that the spacetime coordinates x^{μ} lose their global meanings and have a direct intuitive significance only locally through the physical interpretation of the interval ds. To extract this meaning from (8) at a given spacetime point, we must put it in the same form as (4) at that point.

We shall use the convention whereby repeated Greek indices are summed from 0 to 3 and Latin indices from 1 to 3, indicating spatial indices.

Separating the spatial part in the expression of the interval,[14]

$$
\begin{aligned}
ds^2 = & \quad g_{\mu\nu} dx^{\mu} dx^{\nu} && (9)\\
= & \quad g_{00} \left(dx^0\right)^2 + 2 g_{0k} dx^0 dx^k + g_{ik} dx^i dx^k \\
= & \quad g_{00} \left(\left(dx^0\right)^2 + 2 \frac{g_{0k}}{g_{00}} dx^0 dx^k \right) + g_{ik} dx^i dx^k && (11)\\
= & \quad g_{00} \left(\left(dx^0\right)^2 + 2 \frac{g_{0k}}{g_{00}} dx^0 dx^k \pm \frac{g_{0i} g_{0k}}{g_{00}^2} dx^i dx^k \right) \\
& \quad + g_{ik} dx^i dx^k
\end{aligned}
$$

In (11) we have added and subtracted the same term. Taking the minus sign term outside the parentheses, we are left with a perfect square inside,

$$
\begin{aligned}
ds^2 &= g_{00}\left((dx^0)^2 + 2\frac{g_{0k}}{g_{00}}dx^0dx^k + \frac{g_{0i}g_{0k}}{g_{00}^2}dx^idx^k\right) \\
&\quad -\frac{g_{0i}g_{0k}}{g_{00}}dx^idx^k + g_{ik}dx^idx^k \qquad (12)\\
&= g_{00}\left(dx^0 + \tfrac{g_{0k}}{g_{00}}dx^k\right)^2 - \underbrace{\left(-g_{ik} + \tfrac{g_{0i}g_{0k}}{g_{00}}\right)}_{\gamma_{ik}}dx^idx^k \qquad (13)
\end{aligned}
$$

Expressing (13) in ordinary local inertial coordinates,

$$ds^2 = \quad c^2d\tau^2 - dl^2 \qquad (14)$$

Comparing (13) with (14), the spatial distance is given by,

$$dl^2 = \quad \gamma_{ik}dx^idx^k \qquad (15)$$

and the actual elapsed time measured by physical clocks,

$$d\tau^2 = \quad \frac{g_{00}}{c^2}\left(dx^0 + \frac{g_{0k}}{g_{00}}dx^k\right)^2 \qquad (16)$$

The fact that the elapsed time in (16) contains the spatial terms dx^k is because the difference in the clock readings corresponds to different clocks; one at the point x^k and the other at $x^k + dx^k$, and those clocks are not synchronized unless $g_{0k} = 0$ for $k = 1, 2, 3$.[13]

It is also possible to obtain (15) and (16) by exchanging light signals between the spatial positions x^k and $x^k + dx^k$ (cf. Ref. [13]). So, (15) and (16) are not merely mathematical tricks, but acquire direct physical significance through the HL.

When we want the elapsed time in the same spatial point, we must put $dx^k = 0$ in (16),

$$d\tau = \quad \frac{\sqrt{g_{00}}}{c}dx^0 \qquad (17)$$

For a finite time interval marked by a fixed physical clock, we have,

$$\tau_2 - \tau_1 = \quad \int_{x^0_{(1)}}^{x^0_{(2)}} \frac{\sqrt{g_{00}}}{c}dx^0 \qquad (18)$$

Although it is not a standard denomination, we designate this method of extracting physical distances and actual time rates from the general metric (8) the “Local Metric Formalism (LMF).”

Figure 3 shows the relation among special relativity, the HL, the LMF, and general relativity.

We highlight the fact that to implement the LMF in noninertial reference frames, the EEP and gravitation are not required, and only the local application of special relativity, as allowed by the HL, suffices.

The full structure of general relativity is necessary only to find how the mass-energy distribution determines the $g_{\mu\nu}$; however, we particularly remark that the necessity of (8) and its interpretation are exclusive consequences of special relativity and the HL.

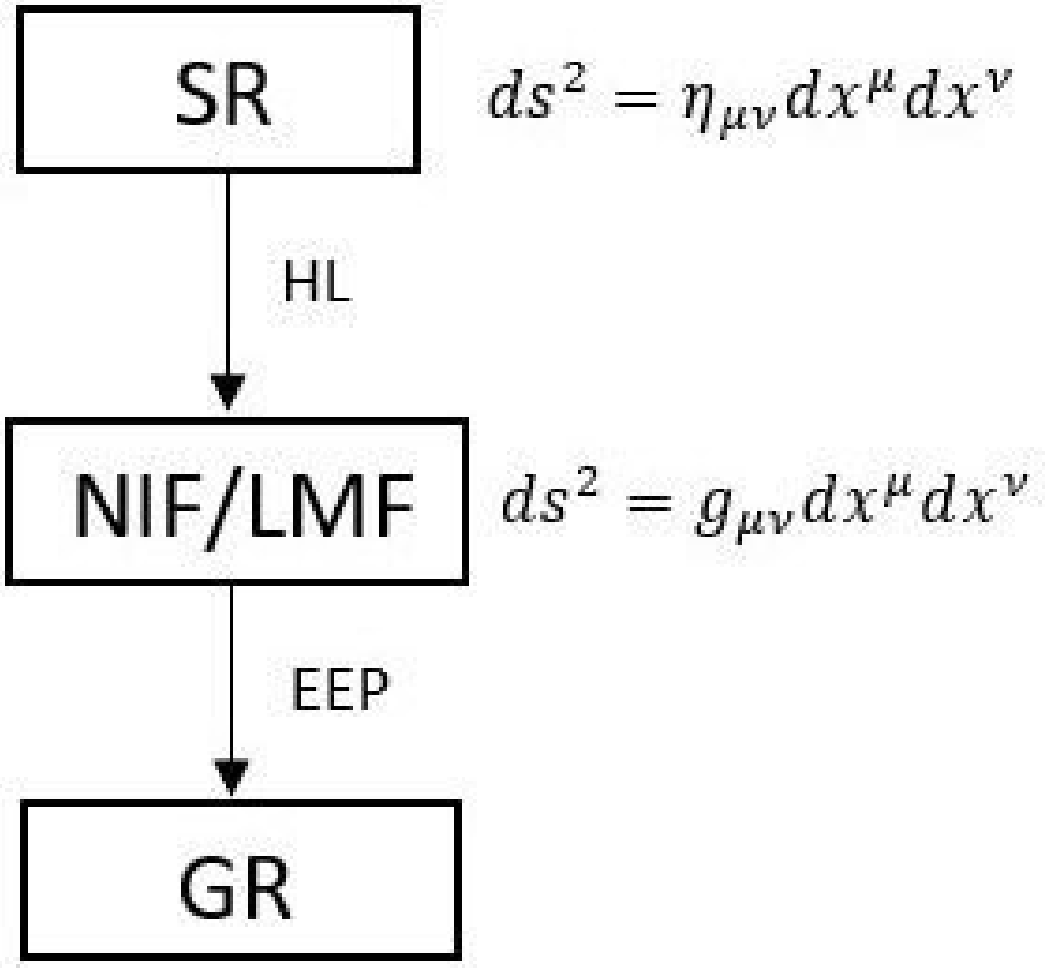


Figure 3. HEURISTIC PATH TO GR. SR ≡ special relativity; HL ≡ hypothesis of locality; NIF ≡ noninertial reference frame; LMF ≡ local metric formalism; EEP ≡ Einstein equivalence principle ; GR ≡ general relativity

V. A Clarifying Example

The incorrect relativistic analysis of rotating noninertial reference frames has led to some erroneous disprovals of special relativity. [4]

One such purported disproof is known as Selleri's paradox,[15][16] and another is the incorrect relativistic analysis of the Sagnac effect.[17][18][19] In both cases, the mistake consists of erroneously assuming global properties of homogeneity and isotropy that are only valid in inertial reference frames and the dismissal of the "only local" validity of special relativity.

A concrete flawed argument against special relativity based on the Sagnac effect claims the following:[17][18]

a. The special relativistic addition of velocities predicts that light speed is invariant for observers fixed along the rim of the rotating platform. (Correct.)
b. Since the clockwise and counterclockwise light beams traverse symmetric paths at the same speeds, an experimenter at C (Fig. 4) rotating with the interferometer must observe both beams arriving simultaneously. Hence, special relativity predicts no displacement of the interference fringes, thus contradicting empirical observation. (Incorrect.)

One counterargument that advocates of relativity often give for rejecting arguments that involve noninertial reference frames, such as the above, is that in those cases, special relativity is not applicable and only general relativity can provide the correct answer.

Although the formalism of general relativity can indeed account for such cases, our point here is to show that special relativity alone suffices and that, from a historical and foundational standpoint, that fact was essential to Einstein for the development of general relativity.

A. The Special Relativistic Solution

For the case that concerns us in this section, i.e., the prediction of the Sagnac effect from within the rotating platform, we transform from the coordinates in the inertial reference frame with origin in the disc center (ct', r', ϕ', z') to general noninertial coordinates fixed in the rotating platform (ct, r, ϕ, z),

$$t = t', r = r', \phi = \phi' - \Omega t, z = z' \tag{19}$$

Here we remark a usual misinterpretation. The time transformation $t = t'$ is sometimes misinterpreted as a Galilean transformation of time, meaning that the variable t inside the platform represents the real time as measured by physical clocks at rest with respect to the rotating platform. [17][18]

However, that is an incorrect interpretation of the time coordinate. When expressed in terms of an arbitrary (noninertial) reference frame, the time coordinate is a mere parameter used to calculate the

actual proper time measured by physical clocks in that frame.

Although according to (19), the time coordinate t can be interpreted as the time of the background inertial reference frame, inside the platform it is only an auxiliary parameter that allows the calculation of the true time indicated by physical clocks attached to the rotating platform according to (17) and (18). This interpretation is not some esoteric effect coming from general relativity but the mere consequence of a consistent “local” application of special relativity.

For the differential interval ds, (19) gives,

$$ds^2 = \underbrace{\left(1-\beta^2\right)}_{g_{00}} c^2 dt^2 - dr^2 - r^2 d\phi^2 - dz^2 - 2\beta r c dt d\phi \tag{20}$$

where $\beta = v_0/c$, $v_0 = r\Omega$. The time elapsed on a physical clock fixed in the same position inside the rotating platform is obtained from (18) and (20),

$$\tau_2 - \tau_1 = \int_{x_1^0}^{x_2^0} \frac{\sqrt{g_{00}}}{c} dx^0 \tag{21}$$

$$\tau_2 - \tau_1 = \int_{t_1}^{t_2} \frac{dt}{\gamma} \tag{22}$$

To evaluate the Sagnac effect, we need to calculate the elapsed times τ^+ and τ^- in the location of the interferometer.

According to the HL, the light paths must satisfy $ds = 0$, which for $r = R$ leads to,

$$dt^+ = +\frac{R d\phi^+}{c\left(1-\beta\right)}, d\phi^+ > 0, (\text{forward beam}) \tag{23}$$

$$dt^- = -\frac{R d\phi^-}{c\left(1+\beta\right)}, d\phi^- < 0, (\text{backward beam}) \tag{24}$$

Replacing (23) in (22),

$$\tau^+ = \int_0^{2\pi} \frac{1}{\gamma} \frac{R d\phi^+}{c(1-\beta)} \tag{25}$$

$$= \frac{1}{\gamma} \frac{L}{c(1-\beta)} \tag{26}$$

Analogously, from (24) and (22),

$$\tau^- = \int_0^{-2\pi} -\frac{1}{\gamma} \frac{R d\phi^-}{c(1+\beta)} \tag{27}$$

$$= \frac{1}{\gamma} \frac{L}{c(1+\beta)} \tag{28}$$

The displacement of the interference fringes is determined by the difference,

$$\tau^{+} - \tau^{-} = \frac{2Lv_0}{c^2\sqrt{1-\left(\frac{v_0}{c}\right)^2}} \tag{29}$$

Putting $L = 2\pi R$ and $v_0 = \omega R$, the last result gives the classical first order value of the Sagnac effect

$$\tau^{+} - \tau^{-} = \frac{4A\omega}{c^2} \tag{30}$$

where $A = \pi R^2$.

That special relativity suffices to explain the Sagnac effect from within the rotating platform is also explained in Ref. [14]

B. Further Clarifications

Although in Section A, we showed how the formal application of the LMF solves the problem of the Sagnac effect according to a consistent local application of special relativity, point b) of the argument may still seem puzzling.

How is it possible that after following symmetrical paths traversed at the same speeds, the two beams do not arrive simultaneously? As we previously observed, in noninertial reference frames, the global homogeneity and isotropy of space, as well as the homogeneity of time, are no longer valid. We can assert that the sole local validity of special relativity in such cases results in even more mind-boggling counterintuitive repercussions than when its global validity applies.

The fact that two observers fixed to the platform, such as A and B (Fig. 4), are instantaneously at rest in different inertial frames that are not themselves at rest with respect to each other has unexpected consequences. Also, in the concrete case of a rotating platform, the rotation has a determinate direction that breaks the symmetry along both paths.

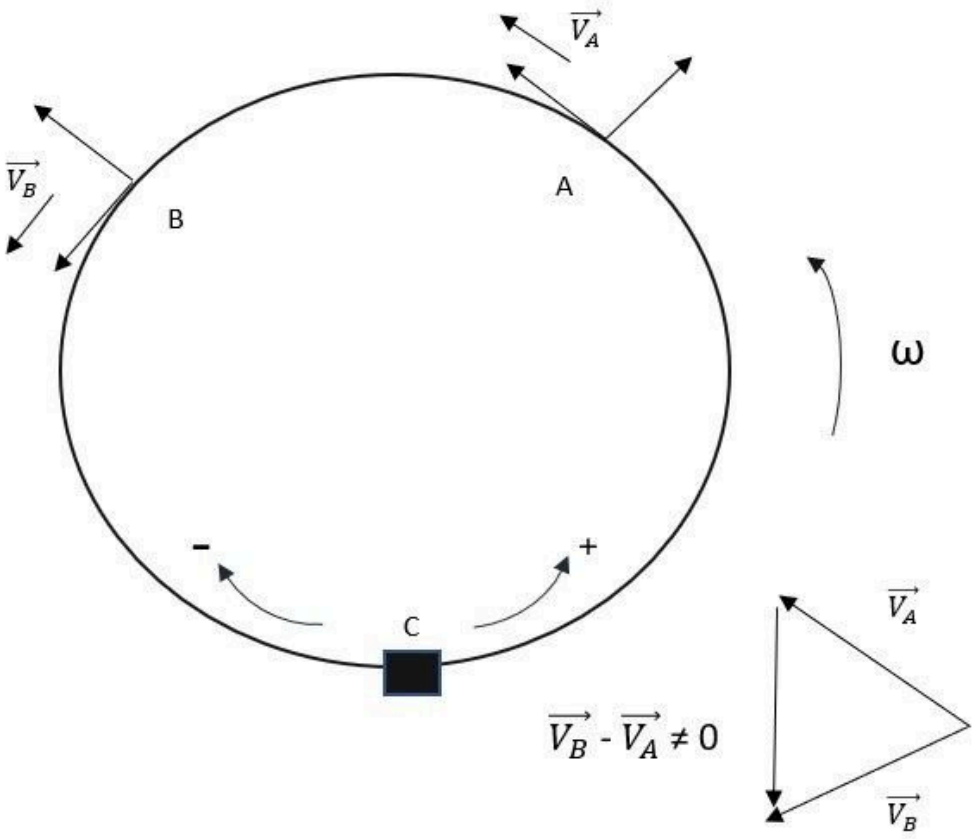


Figure 4. MCIRFs at A and B

Furthermore, the mean speed of the beams calculated with one fixed clock at C does not coincide with the constant speed c measured with local clocks placed along the beam paths. A similar effect also takes place in the gravitational field of the sun. It is known as the Shapiro effect and is considered the fourth classical test of general relativity. [20] This analogy is also mentioned in Ref. [21]

Another usual incorrect interpretation asserts that the local speeds of the beams obtained from (23) and (24) are $c(1-\beta) = c - v$ and $c(1+\beta) = c + v$, respectively. However, that is also incorrect because the coordinates times dt^+ and dt^- are not proper physical times (cf. Ref. [22] for a detailed explanation).

Additionally, the subtlety that the equivalence between acceleration and inertial motion is only local renders the EEP a local principle as well, which cannot be applied globally. The dismissal of this fact can easily lead to incorrect inferences regarding the consistency and validity of the theory of relativity.[23][24]

An instructive and interesting instance discussing similar issues can be found in an exchange between Robert D. Klauber and T. A. Weber. [25][26]

doi.org/10.32388/ZI3ZCD.3

VI. Conclusions

Einstein recognized in 1905 that the equivalence between an accelerated reference frame and a momentarily comoving inertial system constitutes a separate assumption. However, in later applications of the method, Einstein omitted any explicit reference to the assumption and proceeded to apply it systematically without further clarification.

Except for the particular case of the *clock hypothesis*, which some physicists and philosophers admit as an independent hypothesis, [5] the local application of special relativity to accelerated systems of reference has become a self-evident procedure that does not require further clarification. However, since from a strictly logical standpoint:

- An accelerated system is not an inertial one even though its coordinate axes may instantaneously coincide.
- The special theory of relativity is exclusively formulated for relating space and time only between inertial frames of reference.

The ensuing perplexity among nonspecialists arising from applying special relativity to accelerated reference frames is comprehensible.

Thus, either accepted as an independent hypothesis or considered as an evident or even mathematical consequence of the theory, to avoid the confusion revealed by Pepino and Mabile, students should be explicitly warned about the local validity of special relativity in noninertial frames of reference at an early stage of exposure to special relativity.

Statements and Declarations

Funding

No funding for this work

Potential Competing Interests

The author has no conflict of interest to disclose.

Footnotes

[1] Mashhoon's HL should not be confused with the "Principle of locality" stating that an object can be influenced only by its immediate surroundings.

[2] In the modern mathematical formulation of the theory, Einstein's description of the infinitesimal validity of special relativity is expressed by postulating that the tangent space is a Minkowski vector space

[3] In this case, the fact that $g_{\mu\nu} \neq \eta_{\mu\nu}$ does not mean spacetime becomes curved. It means that we are using a noninertial reference frame to describe flat spacetime.

[4] In this section, we are assuming a basic familiarity with the Sagnac effect. Clarifying discussions of the peculiarities of spacetime in rotating reference frames can also be found in Refs. [27][28][25]

[5] The fact that some find the *clock hypothesis* an independent assumption, [4][5][6] while others a consequence of the theory that transforms it into a theorem [29][30] is an example the duality between considering the HL an independent assumption or a natural consequence of the theory.

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Declarations

Funding: No specific funding was received for this work.

Potential competing interests: No potential competing interests to declare.