Review of: "A trial-dependent N-player game"

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Potential competing interests: No potential competing interests to declare.

This paper introduces a new variant of the classical N-player gambler's ruin problem. Here's a breakdown of the key points and some suggestions for improvement:

Strengths:

- Introduces a new variant of the N-player gambler's ruin problem (termed a "game," but it might be more accurately described as a stochastic process).
- Provides clear definitions and formulas for expected fortune and variance.
- Considers both symmetric and asymmetric scenarios.

Areas for Improvement:

- **Genre Clarification:** The title and introduction should reflect that this is a stochastic process, not a game, where players don't make strategic choices. Consider renaming it "A Trial-Dependent N-Player Ruin Model."
- **Problem Statement Clarity:** The initial problem description could benefit from greater precision:
  - N agents with initial budgets participate.
  - In each round, one agent is chosen randomly (probability may depend on the agent) to receive $1 from each other agent (total gain: N-1 dollars).
  - The process ends when an agent reaches zero budget (bankruptcy).
- **Group Distinction:** The paper divides players based on initial budgets. The analytical purpose behind this distinction needs clarification. Alternatively, explore alternative groupings or eliminate it if not contributing significantly.
- **New Process (Θ):** The relationship between the original process and the new, fixed-duration (α) process (Θ) needs better explanation.
  - Does the budget only affect the original process termination time?
  - Can Θ be better understood as a separate random walk with known properties?
- **Proposition Justification:** The paper should provide proofs or justifications for Propositions 1 and 2. Without them, the validity and connection to the model are unclear.
- **Originality and Contribution:** The results might be well-known in existing research on gambler's ruin with stopping times (e.g., cover time). The authors should:

- Carefully assess the novelty of their results.
- If the results are known, acknowledge this and focus on the contribution of the analysis (e.g., applying the model to a specific scenario not previously considered).

- **Presentation:**
  - The introduction's numerous formulas could benefit from explanations about what they represent.
  - Consider replacing tables with graphs for a more informative numerical investigation.
  - Refine notation for clarity (e.g., explain Ai) and avoid introducing and abandoning symbols (X, Θ).
  - Address any grammatical errors.

Additional Considerations:

- Briefly discuss limitations (e.g., lack of closed-form solutions for asymmetric cases) and potential future research directions (e.g., approximation methods).

Conclusion:

This paper presents a new variant of the gambler's ruin problem. By addressing the areas for improvement outlined above, the authors can significantly enhance the clarity, impact, and potential future directions of their research. If the results are indeed well-known, consider refocusing the paper on the application of the model to a specific scenario or exploring extensions with more complexity.