

Review of: "A Mathematical Contradiction in the Special Theory of Relativity"

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Potential competing interests: No potential competing interests to declare.

Review of the paper: A mathematical contradiction in the special theory of relativity, by Alain Haraux.

Reviewer's name: Harish Parthasarathy.

The author considers an inertial frame \$K\$ (the rest frame) and a frame \$K'\$ moving relative to \$K\$ with a uniform velocity \$-v\$ along the x axis, or equivalently with velocity \$v\$ along the negative x axis. \$x,x'\$ denote the x coordinates in \$K,K'\$ respectively.A light source \$S\$ is located at the origin of the rest frame \$K\$ and emits a photon which travels with velocity \$c\$ relative to \$K\$. \$t\$ denotes time as measured in \$K\$ and \$\tau\$ denotes time as measured in \$K'\$. An object \$O\$ is at time \$t=0\$ located at \$x=D\$ and is at rest in the moving frame \$K'\$ After time \$t\$, the object \$O\$ is at \$x=(D-vt)\$ and hence its distance from the photon as seen in \$K\$ at time \$t\$ is \$ct-(D-vt)=(c+v)t-D\$ and therefore as seen in \$K\$, the photon hits the object at time \$t=D/(c+v)\$. On the other hand, since times \$\tau\$ and \$t\$ in \$K'\$ and \$K\$ respectively are related by the time dilation formula \$\tau=t\gamma, \gamma=1\sqrt{1-v^2/c^2}\$ and lengths in \$K',K\$ are related by the length contraction formula \$I'=I/gamma\$, it follows that at time \$t=0\$, the object \$O\$ as seen in \$K'\$ is at located at \$x'=D\gamma\$ and at all times \$\tau\$ in \$K'\$ is located at \$x'=D\gamma\$ since it is at rest in \$K'\$. On the other hand, the photon at time \$\tau\$ in \$K'\$ is located at\$x'=c\tau\$ since the speed of light in \$K'\$ is also \$c\$ by Einstein's postulate in special relativity. Hence, as seen in \$K'\$, the distance between the photon and the object at time \$\tau\$ is given by \$c\tau-D\gamma\$ which means that the photon hits the object at time\$\tau=D\gamma c\$ as seen in \$K'\$ or equivalently, as seen in \$K\$, by time dilation, at time \$t=\tau\gamma==D/c\$. Thus the two methods of evaluating the hitting time of the photon on the object are different, in one case, \$D/(c+v)\$ and in the other case \$D/c\$. This argument has certain flaws which I propose to point out here. Firstly, if time in \$K'\$ gets dilated as compared to time in \$K\$ due to their relative motion, then time in \$K\$ should equivalently get dilated as compared to time in \$K'\$ since only relative motion counts. Actually, \$\tau=t\gamma\$ measures time in \$K'\$ as recorded by a frame attached to the moving object at its origin. In other words, \$\tau\$ is the proper time as measured by a clock attached to any point at rest in \$K'\$. It cannot be used to calculate the times of all events in \$K'\$. To do so, we must actually use \$t',x'\$ rather than \$t,x\$ related by the Lorentz transformation formulae

$$t^{'}=\gamma(t-vx/c^{2}), x^{'}=\gamma(x-vt)$$

It should be noted that both \$K\$ and \$K'\$ are inertial frames in their own right and if time gets dilated by a clock attached to one point in \$K'\$ relative to \$K\$, then so must be the case with \$K\$ and \$K'\$ interchanged. Otherwise, we are in the



situation of the twin paradox. If we take recourse to the general theory of relativity, then an inertial frame is that which is far removed from all matter in the universe and is at rest relative to this far removed matter. It is only relative to such an absolute inertial frame, that other inertial frames can be defined. This notion was introduced by Ernst Mach and is at the heart of Einstein's general theory of relativity. Secondly, Einstein's formula for the relative distance and hence relative velocities between two objects, one the photon and the other the moving object has been abandoned here by stating that the relative distance between the object and the photon as seen in \$K\$ is \$ct-(D-vt)=(c+v)t-D\$. This formula implicitly is based on the assumption that as seen in \$K\$, the relative velocity between the photon and the object is \$c+v\$ which is incorrect, ie, not compatible with the Lorentz transformation. Einstein's formula for the relative velocity between two objects moving with velocities \$u,-v\$, based on the Lorentz transformation is \$(u+v)/(1+uv/c^2)\$ which when we take \$u=c\$ becomes \$c\$ rather than \$c+v\$. I hope, this clarifies the apparent contradiction stated by the author.\end{document}