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# Design of quantum gates using quantum scattering theory

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**Abstract**

Given two Hamiltonians $H_0, H_1 = H_0 + V$ corresponding respectively to a free particle Hamiltonian and the free particle Hamiltonian plus its interaction energy with a scattering centre, we can compute the S-matrix, i.e., the matrix that determines the quantum mechanical amplitude of the process involving a particle coming from a free particle state at an infinite distance from the scattering centre at time $t = -\infty$, entering into an initial scattered state, interacting with the scattering centre, entering into a final scattered state, and then escaping away to infinity at time $t \to \infty$ to a final free particle state. Such amplitudes were first studied by Lippmann and Schwinger (Steven Weinberg, The Quantum Theory of Fields, Vol. 1, W.O. Amrein, Hilbert Space Methods in Quantum Mechanics) and then analyzed in a mathematically precise way by [T. Kato], [W. Amrein], and [K.B. Sinha]. In this paper, we first derive the Lippmann-Schwinger equation for the input and output scattered states in terms of the input and output free particle states using intuitive arguments originally due to Lippmann and Schwinger, and then, using rigorous operator theoretic arguments revolving around the spectral theorem for unbounded self-adjoint operators in a Hilbert space, we obtain an elegant formula for the scattering matrix elements in terms of the interaction potential and the free particle Hamiltonian between two free particle states corresponding to the same measure. We then discuss a computationally more efficient method based on the Dyson series expansion (Steven Weinberg, The Quantum Theory of Fields, Vol. 1), which works even in the case when the interaction Hamiltonian is time varying, in contrast to the Lippmann-Schwinger method, for calculating the S-matrix. We assume that the scattering potential, which can be time varying, can be controlled by incorporating control parameters and explain how the resulting unitary S-matrix can be made to approximate a given unitary gate in infinite-dimensional Hilbert space by optimizing the Hilbert-Schmidt/Frobenius norm distance

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between the given gate and the S-matrix gate, in which the latter is approximated by a truncated Dyson series. Of course, the S-matrix given by the Dyson series is a Taylor series functional expansion in the interaction potential and hence is a highly nonlinear function of the control parameters, and hence we propose numerical methods for such optimization. We then observe that if the control parameters are taken as random variables with joint moments, then the TPCP map involving statistical averaging of the adjoint action of the S-matrix acting on an initial mixed state w.r.t. the probability distribution of the control parameters will result in a final mixed state that is linear in the statistical moments of the control parameters, and hence optimization w.r.t. these moments can be easily carried out. Our method of TPCP map design is to optimize the sum of Frobenius distance squares between a sequence of desired output mixed states and the S-matrix based formula for the TPCP map acting on the corresponding input mixed states w.r.t. the statistical moments of the control parameters. We then generalize this method to include the case of TPCP maps obtained from the Hudson-Parthasarathy quantum noisy Schrödinger equation. In summary, we simulate the S-matrix gate for independent realizations of the control parameters having a probability distribution defined by their joint moments obtained by the above optimization procedure and apply the adjoint action of these independent S-matrices on a given input state, and take their ensemble average to obtain a good approximation to the action of the desired TPCP map on any given input state. This suggests that TPCP maps of arbitrarily large size can be realized using scattering experiments in the laboratory.

# 1 Introduction

We first focus on quantum gate design using quantum scattering theory. The design of unitary gates or TPCP maps using quantum scattering theory experiments enables us to design gates of very large size to implement quantum signal processing operations like the quantum Fourier transform, quantum teleportation of states, quantum phase finding, and quantum order finding and factoring, Grover's search algorithm, etc. The materials required to implement such a quantum gate would involve the purchase of lasers and graphene waveguides to perform ion trap experiments. As a first attempt, we shall use just software to implement the quantum gates and TPCP maps. We then focus on the design of an elementary Cq communication system wherein the transmitter generates a state dependent upon the classical source sequence to be transmitted. This transmitted state gets tensor-coupled to the noisy channel bath, generating quantum noise as per the Hudson-Parthasarthy quantum stochastic calculus, and then the channel noise also gets coupled to the receiver so that the final state of the receiver, given by the partial trace over the transmitter and channel Hilbert spaces of the unitary action of the total system on the initial tensor product state, becomes a function of the source alphabet, which had initially been encoded as the initial transmitter state. The standard methods of Cq coding theory can be applied to decode the source sequence from the received state using detection operators. This study would enable us to simulate real quantum systems using our simplified model. In addition to this, the purchase of the Tenerife quantum computer, based on the interference of photons rather than on electricity, would enable us to incorporate our designed gates into such a computer and thereby speed up the operations of signal processing (The quantum Fourier transform can be performed using $O(n)$ multiplications in contrast to $O(n.log_2n)$ required using a classical FFT).

## 2 Design of quantum gates using quantum scattering theory

**Taken from lecture notes of Harish Parthasarathy, NSUT**

Let $H_0, H = H_0 + V$ be two Hamiltonians. For $|\Phi_\alpha >$ satisfying

$$H_0|\Phi_\alpha >= E(\alpha)|\Phi_\alpha >$$

we define states $|\Psi_\alpha^\pm >$ by the Lippman-Schwinger equations

$$|\Psi_\alpha^\pm >= |\Phi_\alpha > +(E_\alpha - H_0 \pm i\epsilon)^{-1}V|\Psi_\alpha^\pm >$$

Then we see that

$$(E_\alpha - H_0 - V \pm i\epsilon)|\Psi_\alpha^\pm >= (E_\alpha - H_0 \pm i\epsilon)|\Phi_\alpha >= 0$$

or equivalently,

$$(E_\alpha - H)|\Psi_\alpha^\pm >= 0$$

This suggests to us that even when the states $|\Phi_\alpha >, |\Psi_\alpha^\pm >$ are not normalizable so that they belong respectively to the continuous spectra of $H_0$ and $H$ with the same spectral value $E_\alpha$, we can make sense of this equation. We shall soon see that the states $|\Psi_\alpha^\pm >$ are scattered states arising after interaction from initial or final states $|\Phi_\alpha >$ without interaction. Here, $H_0$ is regarded as the free particle Hamiltonian and $H = H_0 + V$ as the Hamiltonian of the particle after interaction with the scattering centre that generates an interaction potential $V$. Indeed, let $|\Phi >$ be an initial non-interacting state, so that it evolves as $|\Phi(t) >= exp(-itH_0)|\Phi >$ and $|\Psi^\in >$ as the scattered state into which the former evolves after scattering, so that it evolves as $|\Psi^{in}(t) >= exp(-itH)|\Psi^{in} >$. The superscript $in$ signifies that as $t \to \infty$, the states $|\Phi(t) >$ and $|\Psi^{in}(t) >$ converge to each other. It follows then that

$$|\Psi^{in} >= \Omega_-|\Phi >$$

where

$$\Omega_- = s.lim_{t \to -\infty} exp(itH).exp(-itH_0)$$

on an appropriate dense domain $D_-$ of the underlying Hilbert space $\mathcal{H}$. Typically, $|\Phi>$ will belong to the continuous spectrum of the isometry $\Omega_-$. Note that $\Omega_-^*\Omega_- = I_{D_-}$ while $\Omega_-\Omega_-^* = P_-$ is the orthogonal projection of $\mathcal{H}$ onto $\mathcal{R}(\Omega_-)$ where $I_{D_-}$ equals the identity operator on $D_-$ which extends uniquely into the identity operator on $\mathcal{H}$ because the former is dense in the latter and the identity operator is a bounded operator.

Likewise, we define $|\Psi^{out}>$ as a scattered state evolving according to $|\Psi^{out}(t)>= exp(-itH)|\Psi^{out}>$ into the free particle state $|\Phi(t)>= exp(-itH_0)|\Phi>$ as $t \to \infty$ Therefore,

$$|\Psi^{out}>= \Omega_+|\Phi>$$

where

$$\Omega_+ = s.lim_{t\to+\infty} exp(itH).exp(-itH_0)|\Phi>$$

again on a dense domain of $\mathcal{H}$ (By domain, we mean linear submanfold). Again $\Omega_+^*\Omega_+ = I_{D_+}$ while $\Omega_+\Omega_+^* = P_+$ where $P_+$ is the orthogonal projection onto $\mathcal{R}(\Omega_+) = \mathcal{R}(P_+)$, the domain of $\Omega_+$ while $I_{D_+}$ is the identity operator on $D_+$ which uniquely extends to the identity operator on $\mathcal{H}$. It should be noted that $\Omega_-$ maps the continuous spectrum of $H_0$ into the continuous spectrum of $H$. Indeed, let $E_0$ denote the spectral measure of $H_0$ and $E$ that of $H$. Suppose $|f>$ belongs to the continuous spectrum of $H_0$. Then, the measure $B \to< f|E_0(B)|f>=\| E_0(B)|f>\|^2$ is absolutely continuous w.r.t the Lebesgue measure on $\mathbb{R}$. But then since

$$\Omega_- exp(isH_0)|f>= exp(isH)\Omega_-|f>, s \in \mathbb{R}$$

it follows that for any Borel subset $B$ of $\mathbb{R}$,

$$\Omega_- E_0(B)|f>= E(B)\Omega_-|f>$$

which implies since $\Omega_-$ is an isometry,

$$\| E_0(B)|f>\|^2=\| \Omega_- E_0(B)|f>\|^2=\| E(B)\Omega_-|f>\|^2$$

and therefore the measure $B \to\| E(B)\Omega_-|f>\|^2$ is absolutely continuous w.r.t the Lebesgue measure, proving the claim. The same is true for $\Omega_+$. We now consider the Lippman-Schwinger equation in the form

$$\int g(\alpha)exp(-iE_\alpha t)\Psi_\alpha^\pm > d\alpha =$$

$$\int g(\alpha)exp(-iE_\alpha t)|\Phi_\alpha > d\alpha + \int g(\alpha)exp(-iE_\alpha t)(E_\alpha - H_0 \pm i\epsilon)^{-1}V|\Psi_\alpha^\pm > d\alpha$$

$$= \int g(\alpha)exp(-iE_\alpha t)|\Phi_\alpha > d\alpha + \int g(\alpha)exp(-iE_\alpha t)(E_\alpha - E_\beta \pm i\epsilon)^{-1}|\Phi_\beta >< \Phi_\beta|V|\Psi_\alpha^\pm > d\alpha.d\beta$$

Taking the plus sign in this equation, we observe that the integrand on the rhs has a pole for the $\alpha$-integral at $E_\alpha = E_\beta - i\epsilon$, in the lower half complex plane and further as $t \to -\infty$ the $\alpha$-integral can be closed by extending the integral to the upper half infinite semicircle since if $Im(E_\alpha) > 0$, then $Re(-iE_{\alpha]}t) \to -\infty$ and the upper half semicircle will thus contribute zero in this limit. Further the $\alpha$-integral over the upper half closed semicircle will contribute zero since as just observed, the poles of the integrand are all in the lower half complex plane. Thus, we get

$$lim_{t\to+\infty}\int g(\alpha)exp(-iE_\alpha t)\Psi_\alpha^+ > d\alpha =$$

$$lim_{t\to+\infty} intg(\alpha)exp(-iE_\alpha t)|\Phi_\alpha > d\alpha$$

which proves that $|\Psi_\alpha^+(t) >$ evolves to $|\Phi_\alpha(t) >$ as $t \to +\infty$. Likewise, it is proven that

$$lim_{t\to-\infty}\int g(\alpha)exp(-iE_\alpha t)|\Psi_\alpha^+ > d\alpha =$$

$$lim_{t\to-\infty}\int g(\alpha)exp(-iE_\alpha t)|\Phi_\alpha > d\alpha$$

and hence that $|\Psi_\alpha^-(t) >$ evolves to $|\Phi_\alpha(t) >$ as $t \to -\infty$. This means that

$$|\Psi_\alpha^+ >= \Omega_+|\Phi_\alpha >, |\Psi_\alpha^- >= \Omega_-|\Phi_\alpha >$$

The picture is therefore that the free particle state $|\Phi_\alpha >$ having energy $E_\alpha$ starting at time $t = -\infty$ evolves to the in-scattered state $|\Psi_\alpha^- >$ and this state in turn gets scattered to the out-scattered state $|\Psi_\beta^+ >$ which then evolves as $t \to +\infty$ to the out-scattered state $|\Phi_\beta >$ with the amplitude for this process being given by

$$< \Psi_\beta^+|\Psi_\alpha^- >=< \Phi_\beta|\Omega_+^*\Omega_-|\Phi_\alpha >$$

$$=< \Phi_\beta|S|\Phi_\alpha >= S(\beta,\alpha)$$

$S = \Omega_+^*\Omega_-$ is the scattering matrix and it is a unitary matrix on the continuous spectrum of $H_0$ and can therefore be extended to a unitary matrix on the entire Hilbert space $\mathcal{H}$. $S(\beta,\alpha)$ defines the representation of the scattering matrix w.r.t the free particle scattered states.

# 3 A method for calculating the matrix elements of $S$ relative to the spectrum of the free Hamiltonian $H_0$

We have

$$\Omega_+ = I + \int_0^\infty \frac{d}{dt}(exp(itH).exp(-itH_0)))dt$$

$$= I + i\int_0^\infty exp(itH).V.exp(-itH_0)dt$$

$$\Omega_- = I - \int_{-\infty}^{0} \frac{d}{dt}(exp(itH).exp(-itH_0))dt$$
$$= I - i\int_{-\infty}^{0} exp(itH).V.exp(-itH_0)dt$$
$$= I - i\int_{0}^{\infty} exp(-itH).V.exp(itH_0)dt$$

Thus,

$$S - I = \omega_+^*\Omega_- - \omega_-^*\Omega_-$$

and

$$\Omega_+^* = I - i\int_0^\infty exp(itH_0).V.exp(-itH)dt,$$
$$\Omega_-^* = I + i\int_0^\infty exp(-itH_0).V.exp(itH)dt$$

so that using

$$exp(-itH)\Omega_- = \Omega_-.exp(-itH_0),$$

we get

$$S = \Omega_+^*\Omega_- = \Omega_- - i\int_0^\infty exp(itH_0).V.\Omega_-.exp(-itH_0)dt$$
$$I = \Omega_-^*\Omega_- = \Omega_- + i\int_0^\infty exp(-itH_0).V.\Omega_-.exp(itH_0)dt$$
$$= \Omega_- + i\int_{-\infty}^{0} exp(itH_0).V.\Omega_-.exp(-itH_0)dt$$

giving finally,

$$R = S - I = -i\int_{-\infty}^{\infty} exp(itH_0).V.\Omega_-.exp(-itH_0)dt$$
$$= -i\int_{-\infty}^{\infty} exp(itH_0)V.exp(-itH_0)dt$$
$$-\int_{t\in\mathbb{R},s\in\mathbb{R}_+} exp(itH_0).V.exp(-isH).V.exp(i(s-t)H_0)dtds$$
$$= -i\int_{-\infty}^{\infty} exp(itH_0)V.exp(-itH_0)dt$$
$$-\int_{t\in\mathbb{R},s\in\mathbb{R}_+} exp(itH_0).V.exp(-isH).V.exp(i(s-t)H_0)dtds$$
$$= -i\int_{\mathbb{R}^3} exp(i(\mu-\lambda)t)dE_0(\lambda).V.dE_0(\mu)dt$$
$$-\int_{\mu,\lambda,t\in\mathbb{R},s\in\mathbb{R}_+} exp(i(\mu-\lambda)t)dE_0(\mu).V.exp(-is(H-\lambda)).V.dE_0(\lambda)dtds$$

$$= -2\pi i \int dE_0(\lambda)V.dE_0(\lambda)/d\lambda$$

$$+2\pi i \int dE_0(\lambda).V.(H-\lambda)^{-1}.V.dE_0(\lambda)/d\lambda$$

Note that here we have made use of the identity

$$\int_{\mathbb{R}} exp(i(\mu-\lambda)t)dt = 2\pi.\delta(\mu-\lambda)$$

Thus, relative to a fixed energy $\lambda$ of the spectrum of $H_0$, the matrix of the operator $R = S - I$ is given by

$$R(\lambda) = -2\pi.i.(V - V.(H-\lambda)^{-1}.V)$$

More precisely, suppose $|\lambda, \alpha >$ denotes an eigenstate of $H_0$ of energy $\lambda$ with the index $\alpha$ running over the joint eigenvalues of a set of commuting variables that along with $H_0$ form a complete set of commuting variables. Then, the matrix of $R$ corresponding to the fixed eigenvalue $\lambda$ of $H_0$ or more precisely to a value $\lambda$ in the spectrum of $H_0$ is given by

$$< \beta|R(\lambda)|\alpha >=$$

$$-2\pi.i[< \lambda,\beta|V|\lambda,\alpha > - < \lambda,\beta|V.(H-\lambda)^{-1}.V|\lambda,\alpha >]$$

$$= -2\pi.i[< \lambda,\beta|V|\lambda,\alpha > - < \lambda,\beta|V.(H_0+V-\lambda)^{-1}.V|\lambda,\alpha >]$$

$$= -2\pi.i[< \lambda,\beta|V|\lambda,\alpha > - \int < \lambda,\beta|V|\lambda',\beta' >< \lambda',\beta'|(H_0+V-\lambda)^{-1}|\lambda'',\alpha' >< \lambda'',\alpha'|V|\lambda,\alpha > d\lambda' d\beta' d\lambda'' d\alpha'$$

In matrix notation, this expression can be expressed as

$$R(\lambda) = -2\pi i.[XVX^* - X.V.(H_0+V-\lambda)^{-1}VX^*]$$

where $X = X(\lambda)$. The problem of gate design is then that we control $V$ by parameters $\theta$ so that $V = V(\theta)$ and choose $\theta$ so that $R(\lambda) = R(\lambda|\theta)$ is as close as possible to a given matrix $R_g = S_g - I = -2\pi iG$, ie we solve the optimization problem

$$argmin_\theta \parallel G - [XV(\theta)X^* - X.V(\theta).(H_0+V(\theta)-\lambda)^{-1}V(\theta)X^*] \parallel^2$$

Specifically, we assume that

$$V(\theta) = V_0 + \sum_{k=1}^{p} \theta[k]V[k]$$

and optimize w.r.t $\{\theta[k]\}_{k=1}^{p}$. Note that in the above expression, $X = X(\lambda)$ is the matrix $Row(< \lambda, \alpha|, \alpha \in F)$ or equivalently, $X^* = Col(|\lambda, \alpha >: \alpha \in F)$ where $F$ is the index set over which the joint eigenvalues or spectral values of all the commuting set of observables except $H_0$ vary.

## 4 Scattering theory for time-varying potentials generated using random control parameters

The case of a time-varying interacting potential. Here, $H(t) = H_0 + V(t)$. We write

$$U_0(t) = exp(-itH_0),$$

$$U(t,s) = T\{exp(-i\int_s^t H(u)du)\}, s \le t$$

It is well known (Dyson series) that

$$U(t,s) = U_0(t).W(t,s)U_0(s)^*$$

where

$$W(t,s) = T\{exp(-i\int_s^t \tilde{V}(u)du)\}$$

with

$$\tilde{V}(t) = U_0(t)^*.V(t).U_0(t) = U_0(-t)V(t)U_0(t)$$

Then,

$$\Omega_+ = lim_{t\to\infty} U(t,0)^* U_0(t),$$

$$\Omega_- = lim_{t\to\infty} U(0,-t)U_0(-t)$$

so

$$S = \Omega_+^* \Omega_- = lim_{t\to\infty} U_0(-t)U(t,-t)U_0(-t)$$

$$= lim_{t\to\infty} W(t,-t) = W(\infty,-\infty)$$

$$= I + \sum_{n\ge 1}(-i)^n \int_{-\infty<t_n<..<t_1<\infty} \tilde{V}(t_1)...\tilde{V}(t_n)dt_1...dt_n$$

Now writing

$$V(t) = \sum_{k=0}^{p} \theta[k]V(k,t)$$

where $\theta(0) = 1$, the problem is to choose $\theta(k), k = 1,2,...,p$ so that $S = S(\theta)$ is as close as possible to a given unitary gate $G_d$, i.e., we solve the optimization problem

$$min_\theta \parallel S(\theta) - G \parallel^2$$

Note that writing

$$\tilde{V}(k,t) = U_0(-t)V(k,t)U_0(t)$$

we can write

$$S(\theta) = I + \sum_{n\ge 1}(-i)^n \sum_{k_1,...,k_n=1}^{p} \theta(k_1),,,\theta(k_n) \int_{-\infty<t_n<...<t_1<\infty} \tilde{V}(k_1,t_1)...\tilde{V}(k_n,t_n)dt_1...dt_n$$

$$= I + \sum_{n\geq 1}(-i)^n(\theta^{\otimes n}\otimes I)^T\int_{-\infty<t_n<...<t_1<\infty}\tilde{V}(t_1)\otimes ...\otimes V(t_n)dt_1...dt_n$$

where the notation is that $\tilde{V}(t_1)\otimes ...\otimes \tilde{V}(t_n)$ is a $p^n\times 1$ vector operator whose $(k_1,...,k_n)^{th}$ entry in lexicographic order is given by $\tilde{V}(k_1,t_1)...\tilde{V}(k_n,t_n)$. Now the optimization problem in gate design can be expressed as

$$min_\theta \parallel I + \sum_{n\geq 1}(\theta^{\otimes n}\otimes I)^T G_n - G_d \parallel^2$$

where

$$G_n = (-i)^n\int_{-\infty<t_n<...<t_1<\infty}\tilde{V}(t_1)\otimes ...\otimes \tilde{V}(t_n)dt_1...dt_n$$

is a vector operator of size $p^n\times 1$. This optimization problem is hard to solve because it is highly nonlinear. So we look at the following statistical problem. Assume $\theta = [\theta[1],...,\theta[p]]^T$ to be a random vector with moments

$$\mu_\theta[n] ==< (\theta^{\otimes n}>, n\geq 1$$

Given an initial mixed state $\rho(i)$, the final state after scattering is

$$\rho(f) =< S(\theta)\rho(i)S(\theta)^* >= \int S(\theta)\rho(i)S(\theta)^* dP(\theta)$$

$$= \sum_{n,m\geq 0,k_1,...,k_n,l_1,...,l_m} < \theta[k_1]...\theta[k_n]\theta[l_1]...\theta[l_m] >$$

$$\times \int \tilde{V}(k_1,t_1)...\tilde{V}(k_n,t_n)\rho(i)\tilde{V}(l_1,s_1)...\tilde{V}(l_m,s_m)dt_1...dt_n ds_1..ds_m$$

$$= \sum_{n,m\geq 0,k_1,...,k_n,l_1,...,l_m} \mu_\theta[k_1,...,k_n,l_1,...,l_m]$$

$$\times \int \tilde{V}(k_1,t_1)...\tilde{V}(k_n,t_n)\rho(i)\tilde{V}(l_1,s_1)...\tilde{V}(l_m,s_m)dt_1...dt_n ds_1..ds_m$$

# 5 Quantum gate design in the presence of quantum noise based on the generalized quantum stochastic calculus of Hudson and Parthasarathy

When noise corrupts the evolution of the particle interacting with the scattering centre, then the free particle unitary dynamics is described as earlier by $U_0(t) = exp(-itH_0)\otimes I_B$, where $I_B$ is the identity operator in the Boson Fock space of the noisy bath, while the perturbed dynamics is described by $U(t)$ which satisfies the H.P.qsde

$$dU(t) = (L_b^a(\theta)d\Lambda_a^b(t))U(t)$$

where

$$L_0^0(\theta) = -iH(\theta), H(\theta) = H_0 + V(\theta),$$

Thus, the scattering matrix in the tensor product of the system and bath Hilbert spaces is given by

$$S(\theta) = I + \sum_{n\geq 1}(-i)^n \int_{-\infty<t_n<...<t_1<\infty} M_{b_1}^{a_1}(\theta, t_1)...M_{b_n}^{a_n}(\theta, t_n)d\Lambda_{a_1}^{b_1}(t_1)...d\Lambda_{a_n}^{b_n}(t_n)$$

where

$$M_b^a(\theta, t) = U_0(-t)L_b^a(\theta, t)U_0(t) = exp(itH_0)L_b^a(\theta, t)exp(-itH_0), a + b \geq 1,$$

$$M_0^a(\theta, t) = \tilde{V}(\theta, t) = exp(itH_0)V(\theta).exp(-itH_0)$$

Remark: Formally, we can consider the time-varying perturbing potential to be $V(\theta) + \sum_{a+b\geq 1} L_b^a(\theta)d\Lambda_a^b(t)/dt$ and then, using the methods of standard time-dependent scattering theory developed above, try to design the parameters $\theta$ in a TPCP map acting on the states of the system Hilbert space with the bath in a coherent state or in a mixture of coherent states given by

$$T(\theta)(\rho_s) = Tr_B(S(\theta)(\rho_s \otimes \rho_B)S(\theta)^*)$$

so that $T(\theta)$ well approximates a given TPCP map $G$ in the sense that for a set of input-output system pair states $(\rho_{1k}, \rho_{2k} = G(\rho_{1k})), k = 1, 2, ..., N$ $T(\theta)$ gives the desired output, i.e., the optimal value of $\theta$ is given by

$$\hat{\theta} = argmin_\theta \sum_{k=1}^{N} \| T(\theta)(\rho_{1k}) - \rho_{2k} \|^2$$

is a minimum.

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